

$$1. \quad y_1 = 20 \log_2 \left(\frac{x_1}{3} \right)^2 \xrightarrow[\text{代入}]{x_1 = 2x_2} y_1 = 20 \log_2 \left(\frac{2x_2}{3} \right)^2 = 20 \left[\log_2 \left(\frac{x_2}{3} \right)^2 + \log_2 4 \right]$$

$$y_2 = 20 \log_2 \left(\frac{x_2}{3} \right)^2 \dots (*)$$

$$\therefore y_1 = 20 \log_2 \left(\frac{x_2}{3} \right)^2 + 40$$

$$(*) \text{ 代入 } \Rightarrow y_1 = y_2 + 40$$

選 (E) #

$$2. \quad f(3) = f(2) - f(1) = [f(1) - f(0)] - f(1) = -f(0) = -\log_2(2-0) = -1, \quad \underline{\text{選 (B)}} \#$$

$$1. \quad \boxed{A} \log_3(x^2 - 3x + 3) < 1 \quad \text{真數} > 0 \Rightarrow x^2 - 3x + 3 > 0 \quad (D = (-3)^2 - 4 \times 1 \times 3 < 0) \Rightarrow x \in \mathbb{R}$$

恒正

$$\text{原式: } \log_3(x^2 - 3x + 3) < \log_3 3 \Rightarrow x^2 - 3x + 3 < 3 \Rightarrow x^2 - 3x < 0 \Rightarrow 0 < x < 3$$

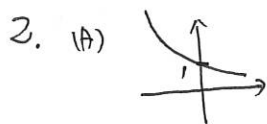
$$A = \{x \mid 0 < x < 3\}$$

$$\boxed{B} \quad -1 \leq \sin(x + \frac{\pi}{2}) \leq 1, \quad B = \{x \mid -\pi < x < \pi\}$$

$$\text{故 } A \cap B = \{x \mid 0 < x < 1\}$$

$$(A) \quad -\frac{\pi}{2} \approx -1.57 \quad (B) \quad \frac{1}{\pi} \approx 0.3 \quad (C) \quad \frac{1}{e} \approx \frac{1}{2.718} = 0.3 \dots \quad (D) \quad \log_9 5 = 0. \dots$$

$$\text{故 } \underline{\text{選 (B)(C)(D)}} \#$$



遞減 (0)



凹口向上 (x)

(C) 正確 (0)

$$(D) \quad \log_a 2018 + \log_a 2020 < 2 \log_a 2019 \Rightarrow \log_a (2018 \cdot 2020) < \log_a 2019^2$$

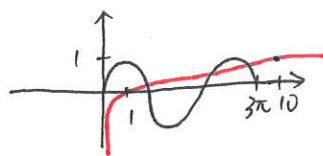
$$\therefore a > 1, \text{ 即 } 2018 \cdot 2020 < 2019^2 \quad (0)$$

$$(E) \quad \text{算幾不等式, } \frac{a^{2018} + a^{2020}}{2} \geq \sqrt{a^{2018} \cdot a^{2020}} = a^{2019}$$

$$\therefore a > 1 \quad (a \neq 1) \quad \therefore a^{2018} \neq a^{2020} \quad \therefore \frac{a^{2018} + a^{2020}}{2} > a^{2019} \quad (0)$$

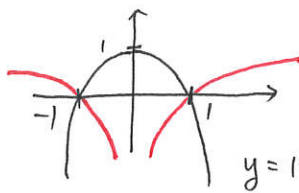
$$\underline{\text{選 (A)(C)(D)(E)}} \#$$

3. (A) $\sin x = |\log x| \Rightarrow \begin{cases} y = \sin x \\ y = \log x \end{cases}$



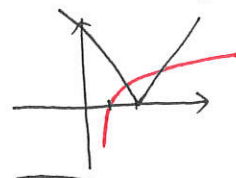
3個實根

(b) $1-x^2 = \log_2 x^2 \Rightarrow \begin{cases} y = 1-x^2 \\ y = 2 \log_2 |x| \end{cases}$
 $= 2 \log_2 |x|$



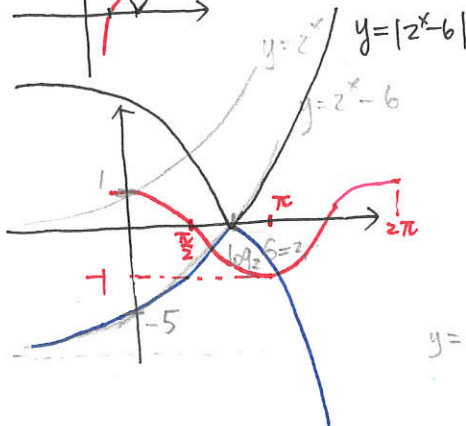
2個實根

(c) $|2-x| = \log x \Rightarrow \begin{cases} y = |2-x| \\ y = \log x \end{cases}$



2個實根

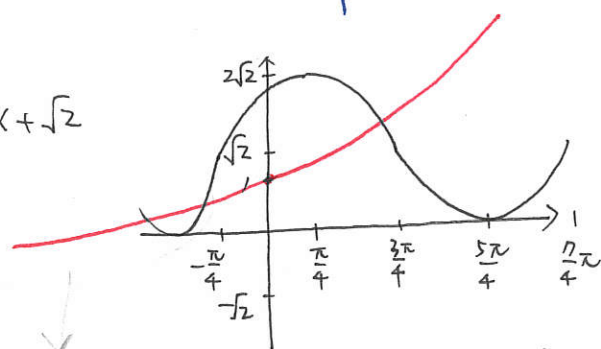
(d) $-|2^x-6| = \cos x \Rightarrow \begin{cases} y = -|2^x-6| \\ y = \cos x \end{cases}$



2個實根

(E) $\sin x + \cos x = \sqrt{2} \sin(x + \frac{\pi}{4})$

$2^x = \sin x + \cos x + \sqrt{2} \Rightarrow \begin{cases} y = 2^x \\ y = \sin x + \cos x + \sqrt{2} \end{cases}$



無限多個交點 \Rightarrow 無限多個實根

選 (B)(C)(D)

三、

1. $(2^x-2)(5^x-3)=0$, $2^x=2$ 或 $5^x=3$, $x=1$ 或 $\log_5 3$

2. $3^{100} \times 2^{50} = (10^{\log_3 3})^{100} \times (10^{\log_2 2})^{50} = 10^{47.71} \times 10^{15.05} = 10^{62.76}$
 $= 10^{0.76} \times 10^{62} = 5. \dots \times 10^{62}$ 故 $a=63, b=5$

$\left(\begin{array}{l} \log 5 \approx 0.699 \\ \log 6 \approx 0.7781 \end{array} \right) \therefore 0.76 = \log 5. \dots \Rightarrow 10^{0.76} = 5. \dots$

$c = 1 \times 4 = 4$

個位數字: $3^1 \Rightarrow 3, 3^2 \Rightarrow 9, 3^3 \Rightarrow 7, 3^4 \Rightarrow 1$ 四次循環 $3^{100} \Rightarrow 3^4 \Rightarrow 1$
 $2^1 \Rightarrow 2, 2^2 \Rightarrow 4, 2^3 \Rightarrow 8, 2^4 \Rightarrow 6$ 四次循環 $2^{50} \Rightarrow 2^2 \Rightarrow 4$

$\therefore 63+5+4 = 72$

$$3. \underbrace{(3^x+5)}_{\text{恒正}} \underbrace{(2^{2x}-16)}_{x=2} \underbrace{(x+\log 10)}_{x=-1} \underbrace{(x+\log \frac{2}{5})}_{x=-\log \frac{2}{5}} < 0$$

$$\begin{array}{c} - \quad + \quad - \quad + \\ | \quad | \quad | \quad | \\ -1 \quad -\log \frac{2}{5} \quad 2 \end{array} \Rightarrow \underline{x < -1 \text{ 或 } -\log \frac{2}{5} < x < \log 2}^*$$

$$4. 2 \leq x \leq 32 \Rightarrow \log_2 2 \leq \log_2 x \leq \log_2 32 \Rightarrow 1 \leq \log_2 x \leq 5$$

$$f(x) = (\log_2 x - 2)^2 - 1$$

$$\therefore \text{當 } \log_2 x = 2 \text{ 時, 有最小值 } N = -1$$

$$\log_2 x = 5 \text{ 時, 有最大值 } M = 8$$

$$\underline{(M, N) = (8, -1)}^*$$

$$5. \textcircled{\text{真}} \text{數} x > 0 \Rightarrow x > 0$$

$$\text{原式: } x^2 - 2x + 3 \Rightarrow (x-3)(x+1) = 0, \quad x = 3 \text{ 或 } -1 \text{ (不合)} \Rightarrow \underline{x = 3}^*$$

$$6. \text{令 } t = 2^x + 2^{-x} \geq 2 \text{ (算幾不等式)}$$

$$t^2 = 4^x + 4^{-x} + 2 \Rightarrow 4^x + 4^{-x} = t^2 - 2$$

$$\text{原式} \Rightarrow 2(t^2 - 2) - 5t + 4 = 0, \quad 2t^2 - 5t = 0, \quad t = 0 \text{ 或 } \frac{5}{2} \Rightarrow t = \frac{5}{2} \text{ (不合)}$$

$$\therefore 2^x + 2^{-x} = \frac{5}{2} \Rightarrow 2(2^x)^2 - 5(2^x) + 2 = 0, \quad (2^x - 2)(2 \cdot 2^x - 1) = 0$$

$$\therefore 2^x = 2 \text{ 或 } \frac{1}{2}, \quad \underline{x = 1 \text{ 或 } -1}^*$$

$$7. \text{A 每天: } 1 \rightarrow 2^{\frac{1}{2}}$$

$$\text{B 每天: } 1 \rightarrow \left(\frac{1}{2}\right)^{\frac{1}{x}}$$

$$\text{(2 天後)} \quad \text{A: } \frac{(2^{\frac{1}{2}})^{12}}{\left[\left(\frac{1}{2}\right)^{\frac{1}{x}}\right]^{12}} = 1024 \Rightarrow 2^{6 + \frac{12}{x}} = 2^{10}, \quad \underline{x = 3}^*$$

$$8. \text{機率和為 } 1 \Rightarrow \log_3 K + \log_9 K + \log_9 \sqrt{K} + \log_{81} K = 1$$

$$\log_3 K + \frac{1}{2} \log_3 K + \frac{1}{4} \log_3 K + \frac{1}{4} \log_3 K = 1$$

$$\geq \log_3 K = 1, \quad \log_3 K = \frac{1}{2}, \quad \underline{K = 3^{\frac{1}{2}} = \sqrt{3}}^*$$

9. $x = \frac{1}{\sqrt{2}+1} = \sqrt{2}-1 \Rightarrow x+1 = \sqrt{2}, x^2+2x-1=0$

$$\begin{array}{r} 1 \quad -1 \\ 1 \quad 1 \quad -3 \quad 3 \\ \hline 1 \quad 2 \quad -1 \\ \quad -1 \quad -2 \quad 3 \\ \quad \quad -1 \quad -2 \quad +1 \\ \hline \quad \quad \quad 2 \end{array}$$

故 $x^3+x^2-3x+3=2$

即求 $\log_8 2 = \frac{1}{3}$

10. $\log x^{2 \log x} = \log \frac{x^3}{10} \Rightarrow (2 \log x)(\log x) = \log x^3 - \log 10$

$\Rightarrow 2(\log x)^2 - 3(\log x) + 1 = 0, (2 \log x - 1)(\log x - 1) = 0.$

$\Rightarrow \log x = \frac{1}{2}$ 或 $1, x = \sqrt{10}$ 或 10 #

IV.

1. 單利 $100 \times (1 + 2.5\% \times 10) = 125$ (萬), $\frac{25}{100} (10)$ #

2. 複利 $100 \times (1 + \frac{2.5\%}{2})^{20} = 100 (1.0125)^{20}$

$(1.0125)^{20} = (10^{\log 1.0125})^{20} = 10^{20 \times 0.00542} = 10^{0.1084} \approx 10^{\log 1.2835} = 1.2835$

$\left(\begin{aligned} \log 1.0125 &= \log \frac{10125}{10000} = \log(3^4 \times 5^3) - \log 10^4 \\ &= 4 \log 3 + 3(1 - \log 2) - 4 \approx 0.00542 \end{aligned} \right)$

\therefore 本利和 $= 100 \times 1.2835 \approx 128.35$ 四捨五入得 128.4 萬元 #