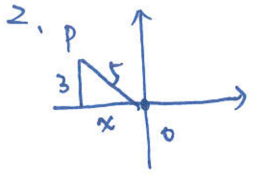
P19. 1. (1) $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$ (2) $\cos 765^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$

(3) $\tan(-150^\circ) = \tan 30^\circ = \frac{\sqrt{3}}{3}$ (4) $\sin 225^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$

(5) $\tan 135^\circ = -\tan 45^\circ = -1$

2. 

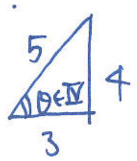
$x = -\sqrt{5^2 - 3^2} = -4$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{-4}{5}$

$\sin\theta + \cos\theta = \frac{3}{5} + \left(\frac{-4}{5}\right) = \frac{-1}{5}$

3. (1) $\sin x = 1 \Rightarrow x = 90^\circ$

(2) $\cos x = \frac{1}{\sqrt{2}} \Rightarrow 45^\circ$, I or IV $\Rightarrow x = 45^\circ$ or 315°

(3) $\tan x = -\frac{1}{\sqrt{3}} \Rightarrow 30^\circ$, II or IV $\Rightarrow x = 150^\circ$ or 330°

4. 

(1) $\sin\theta = -\frac{4}{5}$

(2) $\cos(\theta - 90^\circ) = \cos(90^\circ - \theta) = \sin\theta = \frac{-4}{5}$

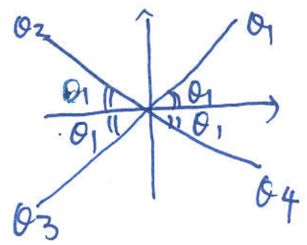
(3) $\tan(180^\circ + \theta) = \tan\theta = \frac{-4}{3}$

5. (1) $(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cdot\cos\theta \Rightarrow \frac{1}{4} = 1 - 2\sin\theta\cos\theta$, $\sin\theta\cos\theta = \frac{3}{8}$

(2) $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cdot\cos\theta = 1 + 2\cdot\frac{3}{8} = \frac{7}{4}$, $\sin\theta + \cos\theta = \pm\frac{\sqrt{7}}{2}$ ($\because \sin\theta\cdot\cos\theta > 0$ 取正) $= \frac{\sqrt{7}}{2}$

(3) $\tan\theta + \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\cdot\sin\theta} = \frac{1}{\sin\theta\cdot\cos\theta} = \frac{8}{3}$

6. $\sin\theta_1 = \sin\theta_2 = \frac{2}{3}$, $\sin\theta_3 = \sin\theta_4 = -\frac{2}{3}$



$\therefore \theta_2 = 180^\circ - \theta_1$

$\theta_3 = 180^\circ + \theta_1$

$\theta_4 = 360^\circ - \theta_1$

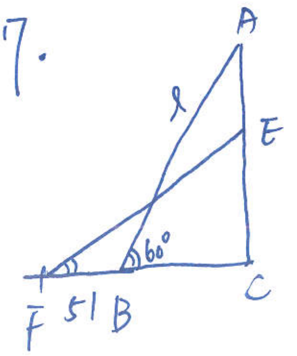
(1) $\theta_1 + \theta_2 = 180^\circ$ (o)

(2) $\theta_2 + \theta_3 = 360^\circ$ (o)

(3) $\theta_1 + \theta_4 = 360^\circ$ (o)

(4) $\sin\theta_2 = \frac{2}{3}$ (o)

(5) $\cos\theta_3 = -\frac{\sqrt{5}}{3}$ (o)

7. 

設梯子長 $\overline{AB} = \overline{EF} = l$

$\because \angle ABC = 60^\circ$ $\therefore \overline{BC} = \frac{1}{2}l$

$\therefore \overline{CF} = 51 + \frac{1}{2}l$

$\because \sin\angle EFC = 0.6 = \frac{3}{5}$ $\therefore \cos\angle EFC = \frac{4}{5} \Rightarrow \frac{51 + \frac{1}{2}l}{l} = \frac{4}{5}$

$\Rightarrow 255 + \frac{5}{2}l = 4l$, $\frac{3}{2}l = 255$, $l = 170$

p22、

1、 $\frac{\pi}{6}$(弧) $= \frac{180°}{6} = 30°$　　$45° = 45\cdot\frac{\pi}{180}$(弧) $= \frac{\pi}{4}$(弧)

$\frac{\pi}{2}$(弧) $= \frac{180°}{2} = 90°$　　$120° = 120\cdot\frac{\pi}{180}$(弧) $= \frac{2}{3}\pi$(弧)

$\frac{3\pi}{4}$(弧) $= \frac{3}{4}\cdot180° = 135°$　　$180° = 180\cdot\frac{\pi}{180}$(弧) $= \pi$(弧)

$\frac{5\pi}{6}$(弧) $= \frac{5}{6}\cdot180° = 150°$　　$270° = 270\cdot\frac{\pi}{180}$(弧) $= \frac{3}{2}\pi$(弧)

2π(弧) $= 2\cdot180° = 360°$

2、 $-\frac{2}{3}\pi$(弧)的同界角為 $-\frac{2}{3}\pi+2k\pi$(弧)，其中 k 為整數.

(2) $k=1$　(3) $k=-1$　(4) $k=2$

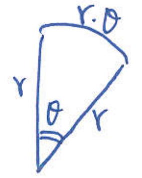
3、 $-\frac{14}{9}\pi$(弧) $= -\frac{14}{9}\times180° = -280°$ 是第一象限角.

4. (1) $\cos\left(\frac{\pi}{4}\right) = \cos45° = \frac{\sqrt{2}}{2}$

(2) $\cos\left(-\frac{11}{6}\pi\right) = \cos(-330°) = \cos30° = \frac{\sqrt{3}}{2}$

(3) $\tan\left(\frac{4}{3}\pi\right) = \tan240° = \tan60° = \sqrt{3}$

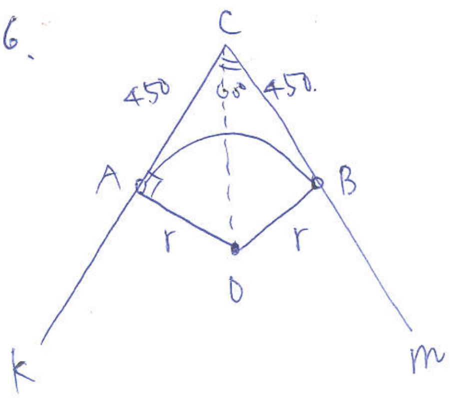
5. 設圓心角 θ，扇形周長 $= 2r+r\theta = 12+6\theta$



圓周長 $= 2\pi r = 12\pi$

$12+6\theta = \frac{1}{2}\cdot(12\pi)$，$6\theta = 6\pi-12$，$\theta = \pi-2$

6、



設 $\overset{\frown}{AB}$ 的圓心 O，半徑 r

∵ $\triangle OAC$ 為 $30°-60°-90°$ 的直角三角形

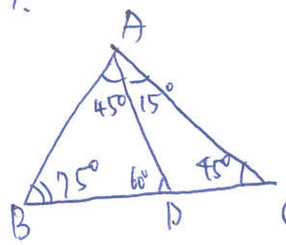
∴ $r = 450\cdot\frac{1}{\sqrt{3}} = 150\sqrt{3}$

又 $\angle AOB = 120° = \frac{2}{3}\pi$

∴ 圓弧長 $= r\theta = 150\sqrt{3}\cdot\frac{2}{3}\pi = 100\sqrt{3}\cdot\pi \approx 544.1944$

(四捨五入後) $= 544$

P26.

1. 

△ABD $\quad \dfrac{\overline{BD}}{\sin 45^\circ} = \dfrac{\overline{AB}}{\sin 60^\circ} \Rightarrow \overline{BD} : \overline{AB} = \sin 45^\circ : \sin 60^\circ = \sqrt{2} : \sqrt{3}$

△ABC $\quad \dfrac{\overline{BC}}{\sin 60^\circ} = \dfrac{\overline{AB}}{\sin 45^\circ} \Rightarrow \overline{BC} : \overline{AB} = \sin 60^\circ : \sin 45^\circ = \sqrt{3} : \sqrt{2}$

$\therefore \overline{BD} : \overline{BC} : (\overline{AB}) = \sqrt{4} : \sqrt{9} : (\sqrt{6}) = 2 : 3 \Rightarrow \overline{BD} : \overline{CD} = 2 : 1$

2. (△ABD) (△ABC)

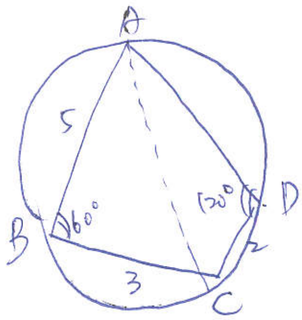
$(\cos B =) \dfrac{10^2+4^2-8^2}{2\cdot 10\cdot 4} = \dfrac{10^2+13^2-a^2}{2\cdot 10\cdot 13} \Rightarrow 13(52) = 4(269-a^2),\ a = 10$

3. (1) (△ABD) (△BCD)

$(\overline{BD}^2 =)\ 6^2+16^2-2\times 6\times 16\times\cos 60^\circ = 6^2+\overline{CD}^2-2\times 6\times\overline{CD}\times\cos 120^\circ$.

$\Rightarrow 256-96 = \overline{CD}^2+6\overline{CD},\ \overline{CD}^2+6\overline{CD}-160=0,\ \overline{CD} = -16$ 或 10
(不合)

(2) ABCD面積 $= \triangle ABD + \triangle BCD = \frac{1}{2}\times 6\times 16\times\sin 60^\circ + \frac{1}{2}\times 6\times 10\times\sin 120^\circ = 39\sqrt{3}$

4. 

(1) (△ABC) (△ACD)
$\overline{AC}^2 = 3^2+5^2-2\times 3\times 5\times\cos 60^\circ = \overline{AD}^2+2^2-2\cdot\overline{AD}\cdot 2\cdot\cos 120^\circ$

(2) $\overline{AC}^2 = 19 = \overline{AD}^2+2\overline{AD}+4,\quad \overline{AD}^2+2\overline{AD}-15=0 \Rightarrow \overline{AD}=3$

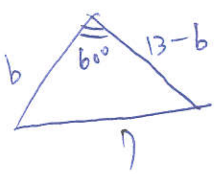
$\overline{AC} = \sqrt{19}$

(3) ABCD面積 $= \triangle ABC + \triangle ACD = \frac{1}{2}\cdot 5\cdot 3\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot 3\cdot 2\cdot\frac{\sqrt{3}}{2} = \frac{21}{4}\sqrt{3}$

(4) $\dfrac{\overline{AC}}{\sin 60^\circ} = 2R,\quad R = \dfrac{\sqrt{19}}{\frac{\sqrt{3}}{2}}\times\dfrac{1}{2} = \dfrac{\sqrt{57}}{3}$. 選(1)(3)(5)

5.

$\dfrac{a}{\sin 60^\circ} = 2R \Rightarrow a = 2\times\dfrac{7\sqrt{3}}{3}\times\dfrac{\sqrt{3}}{2} = 7.\quad b+c=13$, 設 $c = 13-b$



$\cos 60^\circ = \dfrac{b^2+(13-b)^2-7^2}{2\cdot b\cdot(13-b)} = \dfrac{1}{2} \Rightarrow b^2+169-26b+b^2-49 = 13b-b^2$

$\Rightarrow 3b^2-39b+120=0,\ b^2-13b+40=0,\ b = 5$ or 8

$\therefore$ △ABC面積 $= \frac{1}{2}\times 5\times 8\times\sin 60^\circ = r\cdot s \Rightarrow 10\sqrt{3} = r\cdot\frac{20}{2},\quad r = \sqrt{3}$

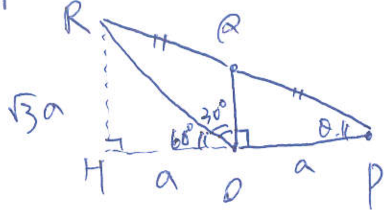
6. (1) $\cos A < 0$, $\angle A$為鈍角 (○) (2) $\sin A = \frac{\sqrt{3}}{2}$, $\angle A$可能為120° (○)

(3) $\angle C > 90^\circ \Rightarrow \cos C < 0 \Rightarrow c^2 > a^2+b^2,\quad \sin^2 C > \sin^2 A+\sin^2 B$ (○)

(4) $a^2 < b^2+c^2 \Rightarrow \angle A$為銳角, 但$\angle B$、$\angle C$可能為鈍角 (×)

(5) $a:b:c = 2:3:4 \Rightarrow$ 最大角為$\angle C \Rightarrow \cos C = \frac{2^2+3^2-4^2}{2\cdot 2\cdot 3} < 0$, $\angle C$為鈍角 (○).

P27. 7.



設 $\overline{OH}=a$, $\overline{RH}=\sqrt{3}a$

$\because \overline{PQ}:\overline{QR}=\overline{PO}:\overline{OH}=1:1$

$\therefore \overline{PO}=a$

$\therefore \tan^2\angle OPQ=\left(\frac{\sqrt{3}a}{2a}\right)^2=\frac{3}{4}$

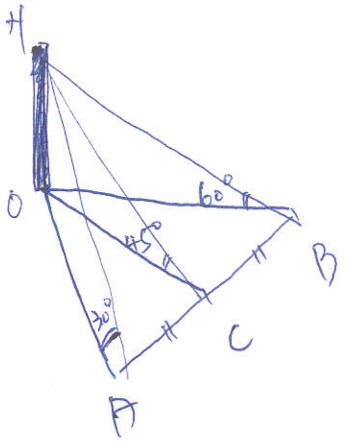
8. 設 $\overline{AP}=a\ (=\overline{RQ}=\overline{RC}=\overline{CQ}) \Rightarrow \overline{AR}=13-a\ (=\overline{PQ}=\overline{BP}=\overline{BQ})$

$\therefore$ APQR 面積 $=a\cdot(13-a)\cdot\sin 60^\circ=20\sqrt{3}$

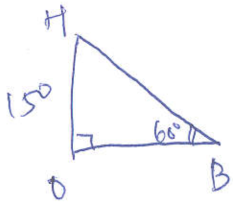
$\Rightarrow 13a-a^2=40,\quad a^2-13a+40=0,\quad a=5$ or 8

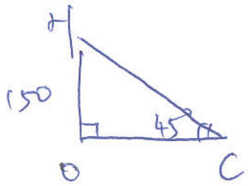
$\therefore \overline{PR}=\sqrt{5^2+8^2-2\cdot5\cdot8\cdot\cos60^\circ}=\sqrt{49}=7$

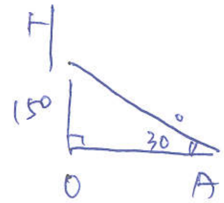
9.



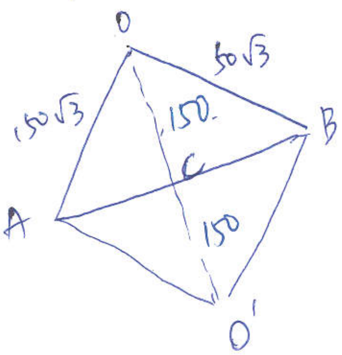
設景觀台最高處 H







$\therefore \overline{OB}=50\sqrt{3}$ $\quad \overline{OC}=150$ $\quad \overline{OA}=150\sqrt{3}$



連接 $\overline{OC}$, 作 O 對 C 之對稱點 O'

$\because \overline{AC}=\overline{CB}$ 且 $\overline{OC}=\overline{CO'}$ $\therefore$ OABO' 為平行四邊形

$\Rightarrow (\overline{OA}^2+\overline{OB}^2)\times 2=\overline{AB}^2+\overline{OO'}^2$

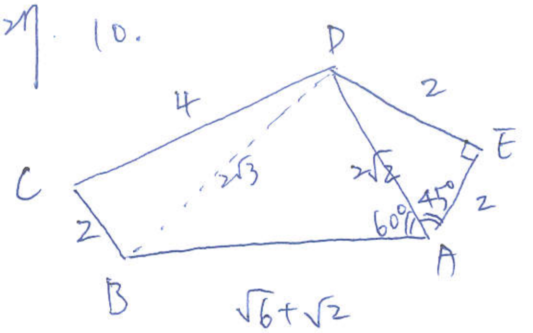
$\Rightarrow \left[(150\sqrt{3})^2+(50\sqrt{3})^2\right]\times 2=\left[(300)^2+\overline{AB}^2\right]$

$\Rightarrow \overline{AB}^2=50^2\times\left[(27+3)\times2-36\right]\Rightarrow \overline{AB}=50\sqrt{24}=100\sqrt{6}.$

$\cos\angle AOB=\dfrac{(150\sqrt{3})^2+(50\sqrt{3})^2-(100\sqrt{6})^2}{2(150\sqrt{3})(50\sqrt{3})}=\dfrac{3^2+1^2-(2\sqrt{2})^2}{2\cdot3\cdot1}=\dfrac{2}{6}=\dfrac{1}{3},\ \sin\angle AOB=\dfrac{2\sqrt{2}}{3}$

$\therefore \triangle AOB$ 面積 $=\frac{1}{2}\cdot150\sqrt{3}\cdot50\sqrt{3}\cdot\frac{2\sqrt{2}}{3}=7500\sqrt{2}$

P31. 10.



(1) $\overline{AD}=\sqrt{2^2+2^2}=2\sqrt2$ (O)

(2) $\angle DAB=105^\circ-45^\circ=60^\circ$ (X)

(3) $\overline{BD}=\sqrt{(2\sqrt2)^2+(\sqrt6+\sqrt2)^2-2(2\sqrt2)(\sqrt6+\sqrt2)\cdot\cos60^\circ}$

$=\sqrt{8+6+4\sqrt3+2-(4\sqrt3+4)}=\sqrt{12}=2\sqrt3$ (X)

(4) $\cos\angle ABD=\dfrac{(\sqrt6+\sqrt2)^2+(2\sqrt3)^2-(2\sqrt2)^2}{2(\sqrt6+\sqrt2)(2\sqrt3)}=\dfrac{12+4\sqrt3}{4\sqrt3(\sqrt6+\sqrt2)}$

$=\dfrac{1}{\sqrt2}$, $\angle ABD=45^\circ$

(5) $\triangle BCD$ 為 $2-2\sqrt3-4$ 即 $1:\sqrt3:2$ 之三角形

∴ $\triangle BCD$面積 $=\frac12\times2\times2\sqrt3=2\sqrt3$ (X)

P32

1. $x\to x+\frac{\pi}{4}$，向左移 $\frac{\pi}{4}$，又因為 $y=\sin x$ 的週期為 2π，
亦可向右移 $\frac{7\pi}{4}$，選 (1)(4) #

2. 振幅為 $a=2$.
週期為 $\frac{2\pi}{b}=\frac{2\pi}{5}\Rightarrow b=5$

3. 振幅為 $a=3$.
週期為 $\frac{2\pi}{b}=8\Rightarrow b=\frac{\pi}{4}$

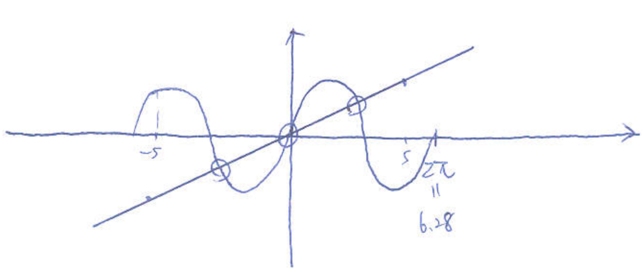
$y=3\sin\left(\frac{\pi}{4}\left(x+\frac{4c}{\pi}\right)\right)$，即原點 (0,0) 向右移 1. ∴ $\frac{4c}{\pi}=-1$. $c=-\frac{\pi}{4}$

4. 刪除，未附圖

5. 週期 $=\frac{2\pi}{100\pi}=\frac{1}{50}$
亦即 1次 $\frac{1}{50}$ 分鐘，故 1分鐘 50次

6. (1) 振幅 $=3$ (2) 週期 $=\frac{2\pi}{\frac{\pi}{2}}=4$ 秒

7. $\begin{cases} y=\sin x \\ y=\frac{x}{5}\end{cases}$



3個交點
∴ 3個實根.