1. 由(b)、(c)、(d)知，共有 $\frac{15+17+18}{2}=25$ 天.

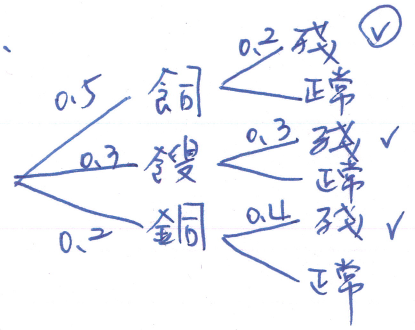
	跑步	游泳	
上午	15		25天
下午	17		25天

∴ 上午游泳 10天
下午游泳 8天

由(a)知：上午游泳則下午跑步 ⇒ 共 10天

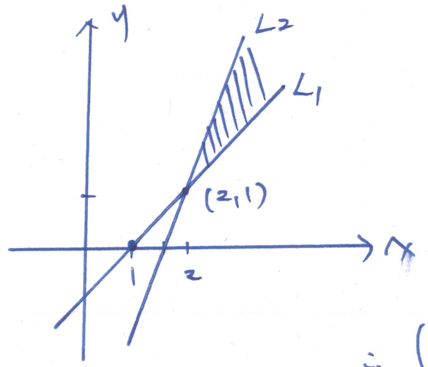
∴ 剩下 17−10=7天下午跑步、上午也是跑步　　(2) #

2.



$$P=\frac{0.5\times0.2}{0.5\times0.2+0.3\times0.3+0.2\times0.4}=\frac{10}{27}\approx0.37$$　　(3) #

3.



繪圖知可行解區域，僅1頂點 (2,1)

∴ $z=ax+by$ 之最小值 $=2a+b=2\sqrt5$

求 a^2+b^2 之最小值.

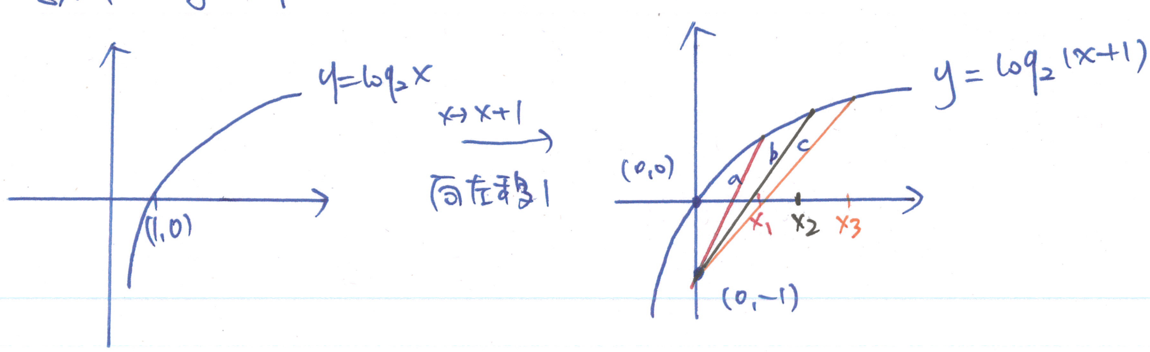
∴ $(a^2+b^2)(2^2+1^2)\geq(2a+b)^2$

$\Rightarrow a^2+b^2\geq\frac{(2\sqrt5)^2}{5}=4$　　(4) #

4.

$$\frac{\log_2(2x+2)}{x}=\frac{\log_2(x+1)+\log_2 2}{x}=\frac{\log_2(x+1)-(-1)}{x-0}$$

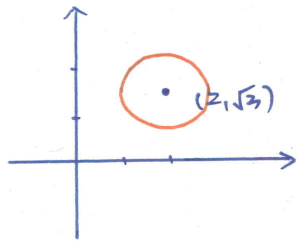
想成 $y=\log_2(x+1)$ 上一點 $P(x,\log_2(x+1))$ 到 $(0,-1)$ 的斜率.



三線段斜率分別為 a, b, c ⇒ $a>b>c$　　(1) #

5、

$$|\vec{OA}+\vec{OB}+\vec{OP}| = |\vec{OA}+\vec{OB}+\vec{OC}+\vec{CP}| = |(2,\sqrt{3})+\vec{CP}|$$

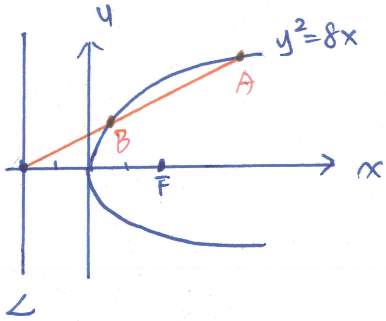


$|\vec{CP}|=1$，表以 C 為圓心之單位圓

∴ P 的最大距離 $= \sqrt{2^2+(\sqrt{3})^2}+1 = \sqrt{7}+1$

(5) ※

6.



∵ $\overline{FA}=2\overline{FB}$

⇒ $d(A,L)=2\times d(B,L)$

設 $A(x_A,y_A)$，$B(x_B,y_B)$

∵ A、B在直線 $y=m(x+2)$ 上　∴ $y_A=m(x_A+2)$

$y_B=m(x_B+2)$

∵ $d(A,L)=2\times d(B,L) \Rightarrow x_A+2=2(x_B+2)$

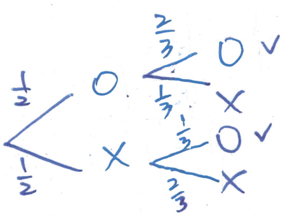
∴ $x_A=2x_B+2$

A、B在Γ上 ⇒ $\begin{cases} [m(2x_B+2+2)]^2=8(2x_B+2) \cdots ① \\ [m(x_B+2)]^2=8x_B \cdots ② \end{cases}$

$\frac{①}{③}$：$\frac{(2x_B+4)^2}{(x_B+2)^2}=\frac{2x_B+2}{x_B} \Rightarrow 4x_B=2x_B+2 \rightarrow x_B=1$，$m=\frac{2\sqrt{2}}{3}$

(4) ※

7. (1)



$\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}=\frac{1}{2}$ (O)

(2) ∵投好、壞球之機率均相等，三振需要3好球，四壞需要4壞球 ⇒ 三振機率高 (O)

(3) 前3球 2好1壞，可能為

OOXO：$\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}$

OXOO：$\frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}$　(X)

XOOO：$\frac{1}{2}\times\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}$

(4) OOO：$\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$ (O)

(5) XXXX：$\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$ (O)

(1)(2)(4)(5) ※

8、

(1) $\mu_{X'} = \frac{40}{10} = 4$ 且 $X' = \frac{X-60}{5}$

$\therefore \mu_{X'} = \frac{\mu_X - 60}{5} \Rightarrow \mu_X = 80$ (○)

(2)

$\sigma_{Y'} = \sqrt{\frac{\Sigma Y'^2}{10} - \left(\frac{10}{10}\right)^2} = \sqrt{1.32-1} = \sqrt{0.32}$

"$\mu_{Y'}$"

$\sigma_{Y'} = \frac{\sigma_Y}{5} \Rightarrow \sqrt{0.32} \times 5 = \sqrt{8} = \sigma_Y$ (×)

(3)

$r_{XY} = r_{X'Y'} = \frac{\Sigma X'Y' - n\mu_X\mu_Y}{\sqrt{\Sigma X'^2 - n\mu_X^2}\sqrt{\Sigma Y'^2 - n\mu_Y^2}} = \frac{41.36 - 10\times4\times1}{\sqrt{160.8-10\times4^2}\sqrt{13.2-10\times1^2}}$

$= \frac{1.36}{\sqrt{0.8}\sqrt{3.2}} = \frac{1.36}{2\times0.8} = 0.85$ (○)

(4)

$\sigma_{X'} = \sqrt{\frac{160.8}{10} - 4^2} = \sqrt{0.08}$, $\sigma_{X'} = \frac{\sigma_X}{5} \Rightarrow \sigma_X = \sqrt{2}$

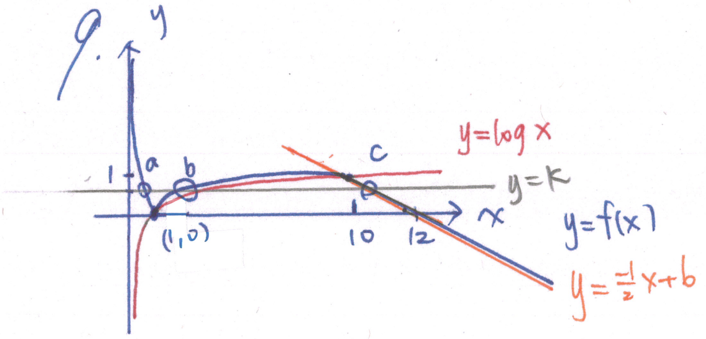
迴歸線： $y - 65 = 0.85 \times \frac{\sqrt{8}}{\sqrt{2}}(x-80)$

$\Rightarrow y - 65 = 1.7(x-80)$ (○)

(5) $y - 65 = 1.7(70-80) \Rightarrow y = 65 - 17 = 48$ (×)

(1)(3)(4) #

9.



設 $f(a)=f(b)=f(c)=k$

由左圖知 $0<k<1$

(1) $-\log a = k \Rightarrow \log a < 0$ (×)

(2) $\log b = k \Rightarrow 0 < \log b < 1$ (×)

(3) $10 < c < 12 \Rightarrow \log c$ 可以很接近 $\log 10 = 1$ (×)

(4) $\log a = -k$, $\log b = k \Rightarrow \log a + \log b = 0 \Rightarrow \log ab = 0 \Rightarrow ab = 1$ (○)

(5) c 可以是 11 (×)

(4) #

10、 $f(2)=a,\ f(3)=b,\ f(-1)=-12,\ f(1)=8$

$\because -1+2i$ 是實係數 $f(x)=0$ 的根 $\Rightarrow$ 有另一根 $-1-2i$

$\therefore f(x)=(x-(-1+2i))(x-(-1-2i))(px+q)$

$=(x^2+2x+5)(px+q)$

$\begin{cases} f(-1)=4(-p+q)=-12 \\ f(1)=8(p+q)=8 \end{cases} \Rightarrow p=2,\ q=-1$

(1)(2) $f(x)=0$ 僅2虛根 $-1\pm 2i \Rightarrow f(2+i)\neq 0,\ f(1+2i)\neq 0$

(3) 偶次項係數和 $=\dfrac{f(1)+f(-1)}{2}=-2$ (×)

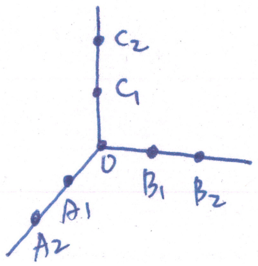
(4) $f(x)=0$ 的解 $-1\pm 2i,\ \dfrac{1}{2}$ (×)

(5) $f(2)=13\cdot 3=0=a.$ $\therefore 2a<b$ (○)

$f(3)=20\cdot(5)=100=b$

(2)(5)※

11、



若選擇的3點中,有2點在同一線上 $\Rightarrow P(V)=0$

$\therefore V=0$ 的方法數 $=\underset{(取一線)}{C^3_1}\underset{(線上2點)}{C^2_2}\times\underset{(其他任一點)}{C^4_1}=12$ 種

共 $C^6_3=20$ 種

若皆選在不同線上

① $A_1、B_1、C_1$: $V=\dfrac{1}{6} \Rightarrow 1$種

② $A_1、B_1、C_2$: $V=\dfrac{1}{6}\times 1\times 1\times 2=\dfrac{2}{6} \Rightarrow 3$種

$A_1、B_2、C_1$

$A_2、B_1、C_1$

③ A_2,B_2,C_1: $V=\dfrac{1}{6}\times 2\times 2\times 1=\dfrac{4}{6} \Rightarrow 3$種

A_2,B_1,C_2

A_1,B_2,C_2

④ $A_2、B_2、C_2$: $V=\dfrac{1}{6}\times 2\times 2\times 2=\dfrac{8}{6} \Rightarrow 1$種

(1) $P(V)$ 可能為 $\dfrac{8}{6}$ (×)

(2) $P(0)=\dfrac{12}{C^6_3}=\dfrac{12}{20}=\dfrac{3}{5}$ (○)

(3) $P(\frac{1}{6})=\dfrac{1}{20}$ (○)

(4) $P(\frac{1}{3})=P(\frac{2}{6})=\dfrac{3}{20}>\dfrac{1}{10}$ (○)

(5) $P(\frac{1}{3})=P(\frac{2}{3})$ (○)

(2)(3)(4)(5)※

12.

(1) $\overline{OP_1}^2 = \left(\sqrt{(10-0)^2+(0-0)^2}\right)^2 = 10^2 = 100$

$S_3 = 100+(100+d)+(100+2d) = 255 \Rightarrow 3d = -45 \Rightarrow d = -15$ (x)

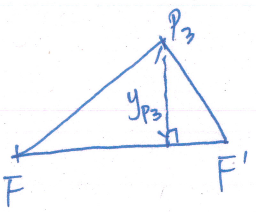
(2) $a_2 = a_1 + d = 100-15 = 85$ (o)

(3) $S_5 = \frac{5(200-60)}{2} = 350$ (x)

(4) $\overline{OP_3}^2 = a_3 = 100-2\times15 = 70 \Rightarrow x_3^2+y_3^2=70$ 又 P_3 在 Γ 上 $\Rightarrow \frac{x_3^2}{100}+\frac{y_3^2}{25}=1$

$\begin{cases} x_3^2+y_3^2=70 \\ x_3^2+4y_3^2=100 \end{cases} \Rightarrow y_3^2=10,\ x_3^2=60$ $\therefore \left|\frac{x_3}{y_3}\right| = \sqrt{6}$ (o)

(5) Γ : $a=10,\ b=5,\ c=5\sqrt{3}$ 由(4)知, $y_3=\sqrt{10}$

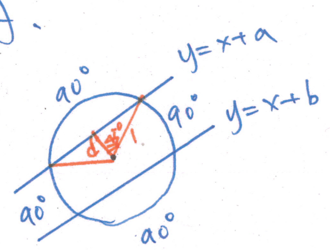


$\triangle P_3FF' = \frac{1}{2}\times \overline{FF'} \times y_{P_3}$

$= \frac{1}{2}\times 10\sqrt{3}\times\sqrt{10} = 5\sqrt{30}$ (o)

(2)(4)(5) #

A.



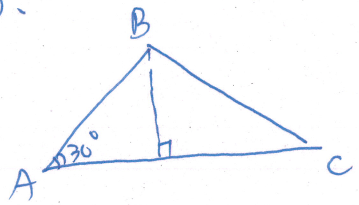
將弧長四等分 ⇒ 每弧度 $=90°$, 圓心角 $=90°$

$\therefore d(O,L) = \frac{1}{\sqrt{2}} = \frac{|0-0+a|}{\sqrt{1^2+(-1)^2}} \Rightarrow |a|=1$

$(45°-45°-90°)$

同理, $|b|=1$, $a^2+b^2=1+1=2$ #

B.

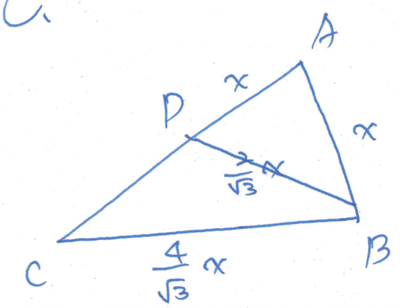


$\overrightarrow{AB}\cdot\overrightarrow{AC} = |\overrightarrow{AB}||\overrightarrow{AC}|\cdot\cos 30° = \tan 30°$

$\therefore |\overrightarrow{AB}||\overrightarrow{AC}| = \frac{1}{\sqrt{3}}\times\frac{2}{\sqrt{3}} = \frac{2}{3}$

$\triangle ABC$面積 $= \frac{1}{2}|\overrightarrow{AB}||\overrightarrow{AC}|\cdot\sin 30° = \frac{1}{2}\times\frac{2}{3}\times\frac{1}{2} = \frac{1}{6}$ #

C.



設 $\overline{AB}=\overline{AD}=x \Rightarrow \overline{BD}=\frac{2}{\sqrt{3}}x$, $\overline{BC}=\frac{4}{\sqrt{3}}x$

由 $\triangle ABC$ 之正弦知: $\frac{\frac{4}{\sqrt{3}}x}{\sin A} = \frac{x}{\sin C}$ ...(1)

又 $\cos A = \frac{x^2+x^2-(\frac{2}{\sqrt{3}}x)^2}{2\cdot x\cdot x} = \frac{1+1-\frac{4}{3}}{2} = \frac{1}{3}$, $\sin A = \frac{2\sqrt{2}}{3}$ ...(2)

(2)代入(1) ⇒ $\frac{\frac{4}{\sqrt{3}}x}{\frac{2\sqrt{2}}{3}} = \frac{x}{\sin C} \Rightarrow \sin C = \frac{2\sqrt{2}}{3}\times\frac{\sqrt{3}}{4} = \frac{\sqrt{6}}{6}$ #

D. 設 $\overline{CD}=h$

$\tan\alpha = \frac{h}{35}$, $\tan\beta = \frac{h}{80}$

∵ $\alpha \geq 2\beta$. tan是遞增 $(0°<\theta<90°)$ ∴ $\tan\alpha \geq \tan(2\beta)$

$\Rightarrow \frac{h}{35} \geq \frac{2\cdot\frac{h}{80}}{1-(\frac{h}{80})^2}$ ∵ $h>0$ 且 $1-(\frac{h}{80})^2>0$ $\Rightarrow \frac{1}{35}\left(1-(\frac{h}{80})^2\right) \geq \frac{1}{40}$

$\Rightarrow 1-(\frac{h}{80})^2 \geq \frac{35}{40} \Rightarrow (\frac{h}{80})^2 \leq \frac{1}{8} \Rightarrow h^2 \leq 80\cdot 10 = 800 \Rightarrow \underline{h \leq 20\sqrt{2}}$ #

E. 設 $\langle a_n \rangle = a_1, a_1+d, a_1+2d, a_1+3d, \dots$

$S_3 = 3a_1+3d = (a_1+d)^2 = a_2^2 \Rightarrow a_1+d = 3 \cdots ①$

$S_1, S_2, S_4 \Rightarrow a_1, 2a_1+d, 4a_1+6d$ 成等比 : $(2a_1+d)^2 = a_1(4a_1+6d) \cdots ②$

① 代入 ② : $(2a_1+3-a_1)^2 = a_1(4a_1+6(3-a_1)) \Rightarrow (a_1+3)^2 = a_1(-2a_1+18)$

∴ $a_1^2+6a_1+9 = -2a_1^2+18a_1 \Rightarrow 3a_1^2-12a_1+9=0 \Rightarrow a_1^2-4a_1+3=0$

$\Rightarrow a_1 = 1$ or 3
- case 1 : $a_1=1$, $d=2$
- case 2 : $a_1=3$, $d=0$ (不合, $d>0$)

∴ $a_{50} = 1+49\times 2 = \underline{99}$ #

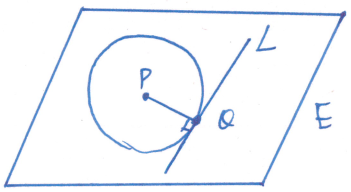
F.

	減		轉		增	
沒有山羊和双魚 :	2	×	2	×	6	= 24
有一張山羊or双魚 :	C^2_1	×			$(C^6_2 + C^6_1 \cdot C^4_1)$	= 78
有二張山羊和双魚	C^2_2	×			C^6_1	= 6

2張增 ; 1張增, 1張其他

$24+78+6 = \underline{108}$ #

G.



$L \perp \overrightarrow{PQ} = (1, 2, -3)$

$L \perp \overrightarrow{n_E} = (1, 1, 1)$

$\cancel{1}\ \ 2\ \ -3\ \ 1\ \ 2\ \ \cancel{-3}$
$\cancel{1}\ \ 1\ \ 1\ \ 1\ \ 1\ \ \cancel{1}$
$5, -4, -1$

$\Rightarrow L \,/\!/\, \overrightarrow{PQ}\times\overrightarrow{n_E} = (5,-4,-1) \,/\!/\, (-5,4,1)$ $\underline{(-5, 4)}$ #

H. $\overrightarrow{T_1T_5}\cdot\overrightarrow{T_1T_9} = (\overrightarrow{OT_5}-\overrightarrow{OT_1})\cdot(\overrightarrow{OT_9}-\overrightarrow{OT_1})$

$= \overrightarrow{OT_5}\cdot\overrightarrow{OT_9} - \overrightarrow{OT_5}\cdot\overrightarrow{OT_1} - \overrightarrow{OT_1}\cdot\overrightarrow{OT_9} + (\overrightarrow{OT_1})^2$

$= \sqrt{2}\cdot\sqrt{2}\cdot\cos 120° - \sqrt{2}\cdot\sqrt{2}\cdot\cos 120° - \sqrt{2}\cdot\sqrt{2}\cdot\cos 120° + \sqrt{2}\cdot\sqrt{2}$

$= \underline{3}$ #