1.

$\begin{cases} x=0,\ \frac{\pi}{2},\ -1,\ 2^{\frac{1}{2}} \\ 但\ \log_2 x\ 有意義 \Rightarrow x>0 \end{cases}$

$\Rightarrow x=\frac{\pi}{2},\ 2^{\frac{1}{2}}$ 其中 $\frac{\pi}{2},\ 2^{\frac{1}{2}}$ 均為無理數 $\Rightarrow$ 0個有理數.

(1) #

2. $B=A+2010$ 由平移.伸縮知.

$C=2A-1$

$D=-10A$

$E=-A+(-500)$

$\sigma_B=\sigma_A$ $\Rightarrow \sigma_A=\sigma_B=\sigma_E<\sigma_C<\sigma_D$

$\sigma_C=2\sigma_A$

$\sigma_D=10\sigma_A$

$\sigma_E=\sigma_A$

(4) #

3. 由根與係數知 $\begin{cases} \alpha+\beta=-a \\ \alpha\beta=b \end{cases}$

(I) $\log\alpha+\log\beta=\log\alpha\beta=\log 6 \Rightarrow \alpha\beta=6$

(II) $2^\alpha\times 2^\beta=2^{\alpha+\beta}=256 \Rightarrow \alpha+\beta=8$

$\therefore a=-8,\ b=6.$ $a-b=-14$

(1) #

4. $2^a,\ 2^b>0.$

$\frac{2^a+2^b}{2}\geq\sqrt{2^a\cdot 2^b}=\sqrt{2^{a+b}}$

"=" 成立 $2^a=2^b\Rightarrow a=b$ $(a,b\in\mathbb{R}$ 即可$)$

(3) $(a^2+a^2)(1^2+1^2)=(a+a)^2$ (o)

(4) $\frac{\log 2^a+\log 2^a}{\log 2}=2a$ (x)

$\log\sqrt{2^{a+a}}=a$

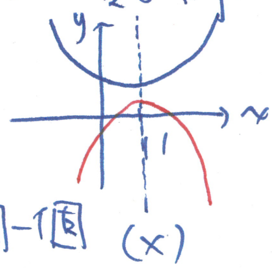
(5) $D=a^2-4a=(a-2)^2-4 \Rightarrow$ 不一定有實數解 (x)

沒有恆 $D>0.$

(3) #

5. 與 x 軸之交點 $\Longleftrightarrow$ 方程式的解（實數解）

2交點　　$(x^2-2x+m)(-x^2+2x+n)=0$　2實數解



$D_1=4-4m$　　$D_2=4+4n$

(1) 若 $m=1 \Rightarrow D_1=0 \Rightarrow$ 1個交點 $x=1$

(2) $\Rightarrow g(x)$ 也有1個交點 $\Rightarrow D_2=0 \Rightarrow x=1$ $\Big]\rightarrow$ 同一個　(×)

(3) 若 $m=2 \Rightarrow D_1<0$
$n=-2 \Rightarrow D_2<0$ $\Big]$ 0個交點 (×)

(4) 若 $m=-1 \Rightarrow D_1=8 \Rightarrow$ 2解 $\dfrac{2\pm\sqrt{8}}{2}$
$n=0 \Rightarrow D_2=4 \Rightarrow$ 2解 $\dfrac{-2\pm\sqrt{4}}{-2}$ $\Big]\rightarrow$ 4個交點 (×)

(5) 若 $m=0 \Rightarrow D_1=4 \Rightarrow$ 2解 $\dfrac{2\pm\sqrt{4}}{2}$
$n=0 \Rightarrow D_2=4 \Rightarrow$ 2解 $\dfrac{-2\pm\sqrt{4}}{-2}$ $\Big]\rightarrow$ 2個交點 (○)

(5)#

6. $(2+i)^6 = 2^6 + C^6_1\cdot 2^5\cdot i + C^6_2\cdot 2^4\cdot \underbrace{i^2}_{=-1} + C^6_3\cdot 2^3\cdot \underbrace{i^3}_{=-i} + C^6_4\cdot 2^2\cdot \underbrace{i^4}_{=1} + C^6_5\cdot 2\cdot \underbrace{i^5}_{=i} + C^6_6\cdot \underbrace{i^6}_{=-1}$

$\therefore b = C^6_1\cdot 2^5 - C^6_3\cdot 2^3 + C^6_5\cdot 2$

(2)#

二、

7. (1) $\sqrt{2014}-\sqrt{2013}=\dfrac{1}{\sqrt{2014}+\sqrt{2013}}$　　$\therefore \dfrac{1}{\sqrt{2014}+\sqrt{2013}} < \dfrac{1}{\sqrt{2013}+\sqrt{2012}}$　(×)

$\sqrt{2013}-\sqrt{2012}=\dfrac{1}{\sqrt{2013}+\sqrt{2012}}$

(2)

$2^{2014}-2^{2013}=2^{2013}\cdot(2-1)$　　(○)

$2^{2013}-2^{2012}=2^{2012}\cdot(2-1)$

(3)

$(0.5)^{2014}-(0.5)^{2013}=(0.5)^{2013}(0.5-1)=$ 負少 $\rightarrow$ 大　(○)

$(0.5)^{2013}-(0.5)^{2012}=(0.5)^{2012}(0.5-1)=$ 負多 $\rightarrow$ 小

(4) $\log 2014-\log 2013=\log\frac{2014}{2013}=\log\left(1+\frac{1}{2013}\right)$ 小、

$\log 2013-\log 2012=\log\frac{2013}{2012}=\log\left(1+\frac{1}{2012}\right)$ 大 (×)

(5) $C^{20}_{14}-C^{20}_{13}=\frac{20!}{14!6!}-\frac{20!}{13!7!}=\frac{20!}{12!6!}\left(\frac{1}{14\times13}-\frac{1}{13\times7}\right)$

$C^{20}_{13}-C^{20}_{12}=\frac{20!}{13!7!}-\frac{20!}{12!8!}=\frac{20!}{12!6!}\left(\frac{1}{13\times7}-\frac{1}{8\times7}\right)$ (○)

$\frac{1}{14\times13}-\frac{1}{13\times7}=\frac{1}{7}\left(\frac{1}{26}-\frac{1}{13}\right)=\frac{1}{7}\times\left(\frac{-1}{26}\right)$ 大

$\frac{1}{13\times7}-\frac{1}{8\times7}=\frac{1}{7}\left(\frac{1}{13}-\frac{1}{8}\right)=\frac{1}{7}\times\left(\frac{-5}{104}\right)$ 小

(2)(3)(5) ※

8. X 平均 $\mu_X=40$，標準差 $\sigma_X>0$

(1) $Y=\frac{4}{5}X+20 \Rightarrow \mu_Y=52$，$\sigma_Y=\frac{4}{5}\sigma_X$

(2) $Z=\frac{5}{4}X+10 \Rightarrow \mu_Z=60$，$\sigma_Z=\frac{5}{4}\sigma_X$ $\Rightarrow \sigma_Z>\sigma_X>\sigma_Y$

(3)(4) Y、X 及 Y、Z 及 Z、X 均為直線關係 $\Rightarrow r_{XY}=r_{YZ}=r_{ZX}=1$

$\left(Y=\frac{4}{5}X+20\right)$ $\left(Z=\frac{5}{4}X+10\right)$

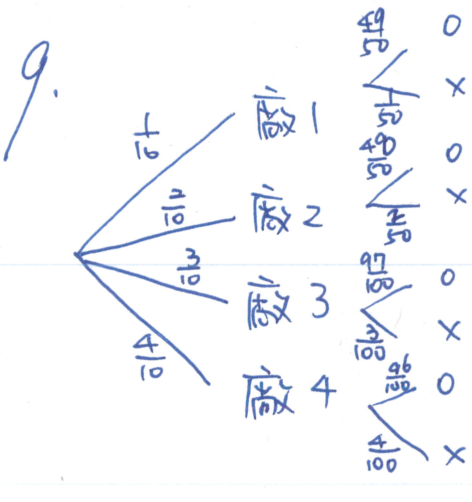
$\Rightarrow X=\frac{5}{4}(Y-20) \Rightarrow Z=\frac{5}{4}\left[\frac{5}{4}(Y-20)\right]+10$

(5) $r_{XY}=1 \Rightarrow r_{X'Y'}=1$ 又 $\mu_{X'}=0$，$\mu_{Y'}=0$，$\sigma_{X'}=1$，$\sigma_{Y'}=1$

$\therefore$ 迴歸線 $y'-0=1\cdot\frac{1}{1}(x'-0) \Rightarrow y'=x'$ (×)

(1)(2)(4) ※

9.



(1) 0.2

(2) $\frac{1}{10}\times\frac{1}{50}+\frac{2}{10}\times\frac{2}{50}+\frac{3}{10}\times\frac{3}{100}+\frac{4}{10}\times\frac{4}{100}=\frac{2+8+9+16}{1000}=\frac{35}{1000}$

(3) $\frac{2}{10}\times\frac{2}{50}=\frac{4}{500}$

(4) $\frac{(4)}{(2)}=\frac{\frac{8}{1000}}{\frac{35}{1000}}=\frac{8}{35}$ (×)

(5) 新廠4 $=\frac{\frac{16}{1000}}{\frac{35}{1000}}=\frac{16}{35}=2\times\frac{8}{35}$ (○)

(1)(2)(5) ※

10、

(1) $6! = 720$ (O)

(2) $\underbrace{3!}_{\text{第一列}} \times \underbrace{3!}_{\text{第二列}} = 36$ (X)

(3) 有順序性 ⇒ 視為相同物

$\square_1$	$\triangle_3$	$\bigcirc_5$
$\square_2$	$\triangle_4$	$\bigcirc_6$

可想成 □□△△○○ 6人排列

$\Rightarrow \dfrac{6!}{2!2!2!} = 90$ (O)

Ex: $\overset{1}{\triangle}\ \overset{2}{\square}\ \overset{3}{\bigcirc}\ \overset{4}{\square}\ \overset{5}{\triangle}\ \overset{6}{\bigcirc}$

2	1	3
4	5	6

(4)

□	□	□
△	△	△

$\dfrac{6!}{3!3!} = 20$ (X)

(5) 和相同 (3行) ⇒ 每行 $= \dfrac{1+2+3+4+5+6}{3} = 7 = 1+6$

$= 2+5$

$= 3+4$

○○○ 放入 (1+6)、(2+5)、(3+4) $\Rightarrow 3! \times \underset{\uparrow}{2}^3 = 48$ (O)

上下對換

(1)(3)(5)

11、 $f(x)=-x^4+ax^3+bx^2+cx+d$

$g(x)=0$ 有一根 $-1+2i$ $\Rightarrow$ 有另一根 $-1-2i$，又 $g(x)$ 領導係數為1

$\Rightarrow g(x)=1\cdot(x-(-1+2i))(x-(-1-2i))$

$=x^2+2x+5=(x+1)^2+4$

$\because h(x)$ 有相同頂點 $\Rightarrow h(x)=a(x+1)^2+4$

又 $f(x)=g(x)\cdot h(x)$，考慮 x^4 係數 $\Rightarrow a=-1$

(1) $g(x)=0$ 僅二根 $-1\pm 2i$ $\Rightarrow g(1+2i)\neq 0$. (×)

(2) $f(x)=(x^2+2x+5)(-(x+1)^2+4)$

各項係數和 $=-1+a+b+c+d=f(1)=8\cdot 0=0$

$\Rightarrow a+b+c+d=1>0$ (o)

(3)

$h(x)=-x^2-2x-1+4=-x^2-2x+3$

$h(x)=0\Rightarrow -(x+3)(x-1)=0 \Rightarrow x=-3$ or 1

$\therefore f(x)=0$ 即 $g(x)=0$ or $h(x)=0 \Rightarrow x=-1\pm 2i, -3, 1$ (o)

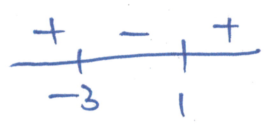
(4)

$h(x)=-(x+1)^2+4$，$-1\le x\le 1$

$\therefore x=1$ 有最小值 $=0$ (×)

(5) $f(x)=\cancel{(x^2+2x+5)}\cdot(-1)(x+3)(x-1)>0$

$\Rightarrow (x+3)(x-1)<0$



$\Rightarrow -3<x<1$ (×)

(2)(3) 𝄞

12、(3) 可發現選項有遞迴關係式

$\Rightarrow$ 走到 K = 走到(k−1)丢正面 + 走到(k−2)丢反面.

$$a_k = a_{k-1} \times \frac{1}{2} + a_{k-2} \times \frac{1}{2} \quad (0)$$

(*)

(1)(2) $a_1 = \frac{1}{2}$ (第1次正)

$a_2 = \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$
(反) (正,正)

$a_1 < a_2$ (O)

$a_2 > a_3$ (X)

$a_3 = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{3}{4} = \frac{5}{8}$
(By (*))

(4)

$$a_k = \frac{1}{2} a_{k-1} + \frac{1}{2} a_{k-2}$$

$$\Rightarrow a_k - a_{k-1} = -\frac{1}{2} a_{k-1} + \frac{1}{2} a_{k-2}$$

$$= \frac{-1}{2}(a_{k-1} - a_{k-2})$$

$\therefore \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}} = \frac{-1}{2} \Rightarrow$ 公比 $= \frac{-1}{2}$ (X)

◎ (5)

$a_3 - a_2 = \frac{-1}{2}(a_2 - a_1)$

$a_4 - a_3 = \frac{-1}{2}(a_3 - a_2)$

$\vdots$

$\times)\ a_{10} - a_9 = \frac{-1}{2}(a_9 - a_8)$

$a_{10} - a_9 = \left(\frac{-1}{2}\right)^8 \cdot (a_2 - a_1)$

同理 $a_3 - a_2 = \left(\frac{-1}{2}\right)(a_2 - a_1)$

$\vdots$

$\times)\ a_9 - a_8 = \left(\frac{-1}{2}\right)(a_8 - a_7)$

$a_9 - a_8 = \left(\frac{-1}{2}\right)^7 (a_2 - a_1)$

依此類推:

$a_2 - a_1 = a_2 - a_1$

$a_3 - a_2 = \left(\frac{-1}{2}\right)(a_2 - a_1)$

$\vdots$

$a_9 - a_8 = \left(\frac{-1}{2}\right)^7 (a_2 - a_1)$

$+)\ a_{10} - a_9 = \left(\frac{-1}{2}\right)^8 (a_2 - a_1)$

$a_{10} - a_1 = \left[1 + \left(\frac{-1}{2}\right) + \cdots + \left(\frac{-1}{2}\right)^8\right](a_2 - a_1)$

$$\therefore a_{10} = \frac{1}{2} + \frac{1\left(1 - \left(\frac{-1}{2}\right)^9\right)}{1 - \left(\frac{-1}{2}\right)} \cdot \left(\frac{3}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{2} + \left(1 + \frac{1}{512}\right) \times \frac{2}{3} \times \frac{1}{4}$$

$$= \frac{1}{2} + \frac{513}{3072} = \frac{2049}{3072}$$

$$= \frac{683}{1024} \quad (0)$$

(1)(3)(5) #

二、

A. $\dfrac{8}{\sqrt{20-2\sqrt{96}}} < n < \dfrac{8}{\sqrt{28-16\sqrt{3}}}$

$\dfrac{8}{\sqrt{20-2\sqrt{96}}} = \dfrac{8}{\sqrt{12}-\sqrt{8}} = \dfrac{8(\sqrt{12}+\sqrt{8})}{4} = 2(\sqrt{12}+\sqrt{8}) \doteqdot 12.\cdots$

(考慮 $[2(\sqrt{12}+\sqrt{8})]^2 = 4(20+2\sqrt{96}) = 4(20+\underset{19\sim20}{\underbrace{\sqrt{384}}}) \approx 156\sim160$)

$\dfrac{8}{\sqrt{28-16\sqrt{3}}} \times \dfrac{\sqrt{28+16\sqrt{3}}}{\sqrt{28+16\sqrt{3}}} = 2\sqrt{28+16\sqrt{3}} = 2\underbrace{\sqrt{28+\sqrt{768}}}_{\sqrt{55}\sim\sqrt{56}} = 14.\cdots$

$7.5^2 = 56.25$

$\therefore 13 \le n \le 14$，但 $\frac{n}{8}$ 為最簡分數 $\Rightarrow$ $n=13$. #

B. $y = \frac{1}{2}x+2 \Rightarrow$ ① 過點 $(\mu_x, \mu_y) = (\frac{5}{2}, \frac{5+a+b}{4})$

② 斜率 $m = r\cdot\frac{\sigma_y}{\sigma_x}$

由①知：$\dfrac{5+a+b}{4} = \dfrac{5}{4}+2 = \dfrac{13}{4} \Rightarrow a+b=8$. #

C. A、B 獨立 $\Rightarrow P(A\cap B) = P(A)\cdot P(B)$

A、C 互斥 $\Rightarrow A\cap C = \phi$，$P(A\cap C) = 0$

$P(A\cup B) = P(A)+P(B)-P(A)\cdot P(B) = P(A\cup C) = P(A)+P(C)-0$

$\therefore P(B)-P(A)\cdot P(B) = P(C)$

$\frac{2}{3} - P(A)\cdot\frac{2}{3} = \frac{1}{2} \Rightarrow \frac{2}{3}P(A) = \frac{1}{6} \Rightarrow P(A) = \frac{1}{4}$ #

D.

$|2x-a|<1 \Leftrightarrow x^2-ax+\frac{3}{4}<0$

$\Downarrow \qquad\qquad \Downarrow$

$4x^2-4ax+a^2<1 \qquad 4x^2-4ax+3<0$

$\Downarrow$

$4x^2-4ax+(a^2-1)<0$

$\therefore a^2-1=3$

$a^2=4 \Rightarrow a=\pm 2$ (取正) $\Rightarrow \underline{a=2}$ #

E.

$(1.5)^{60}=\left(\frac{3}{2}\right)^{60}=\left(\frac{15}{10}\right)^{60}=\frac{15^{60}}{10^{60}}$

分母 10^{60} 表示小數點後有60位.

$15^{60} \Rightarrow \log 15^{60}=60\log 15=60(\log 3+\log 5)=70.566$

15^{60} 共有71位數 $\Rightarrow$ 有60位在小數點後, 有11位在小數點前

$\underline{(11,60)}$ #

F. 設首項a, 公差d

$a+2d, a+5d, a+9d$ 成等差

$\Rightarrow (a+5d)^2=(a+2d)(a+9d)$

$\Rightarrow a^2+10ad+25d^2=a^2+11d+18d^2$

$\Rightarrow ad=7d^2 \Rightarrow d=0$ (不合) or $a=7d$, 又 $a=1 \Rightarrow d=\frac{1}{7}$

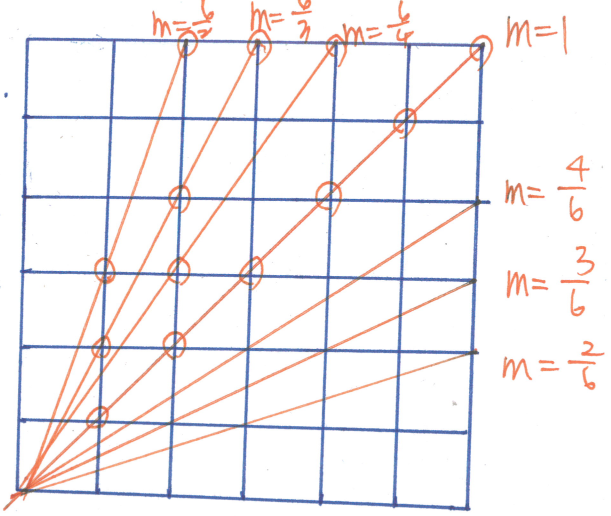
$S_n=\frac{n(2\times 1+(n-1)d)}{2}>10 \Rightarrow n\left(2+\frac{n-1}{7}\right)>20$

$\Rightarrow n(14+n-1)>140 \qquad \therefore n>7$ or $n<-20$

$\Rightarrow n^2+13n-140>0 \qquad$ 取 $\underline{n=8}$ #

$\Rightarrow (n+20)(n-7)>0$

5.



排法 果2互換

$m=1 \Rightarrow C^6_2 \times 2!$

$m=\frac{6}{4}, \frac{4}{6} \Rightarrow C^2_2 \times 2! \times 2$

$m=\frac{6}{3}, \frac{3}{6} \Rightarrow C^3_2 \times 2! \times 2$

$m=\frac{6}{2}, \frac{2}{6} \Rightarrow C^2_2 \times 2! \times 2$

共50種.

$$P=\frac{50}{36\times 36}=\frac{25}{648} \#$$

4.

$$a_k = 3\times\underbrace{2\times 2\times\cdots\times 2}_{(k-1)\text{個}} = 3\times 2^{k-1}$$

$$\sum_{k=1}^{n} a_k = 3+3\times 2+3\times 2^2+\cdots+3\times 2^{n-1}$$

$$=\frac{3(2^n-1)}{2-1} > 10^4$$

$$\Rightarrow 2^n-1 > \frac{10000}{3} = 3xxx$$

$2^{10}=1024,\ 2^{11}=2048,\ 2^{12}=4096$

$\therefore n=12$ #