1. $16:9 = \frac{16}{25}$ or $\frac{9}{25} = 0.64$ or $0.36 \Rightarrow \overline{CD}$ or $\overline{FG}$ (3)#

2. T 的平均 $= \frac{(-3)+1+11+15+x}{5} = \frac{24+x}{5}$

(1) $\frac{24+x}{5} = 12.60 \Rightarrow 24+x = 63 \Rightarrow x = 39$，此時 $S \cap T = \phi$

(2) $\frac{24+x}{5} = 6.2 \Rightarrow 24+x = 31 \Rightarrow x = 7$，此時 $S \cap T = \{7\}$，不合.

(3) $\frac{24+x}{5} = 5.2 \Rightarrow 24+x = 26 \Rightarrow x = 2$，此時 $S \cap T = \phi$

(4) $\frac{24+x}{5} = 4.8 \Rightarrow 24+x = 24 \Rightarrow x = 0$，此時 $S \cap T = \phi$

(5) $\frac{24+x}{5} = 4 \Rightarrow 24+x = 20 \Rightarrow x = (-4)$，此時 $S \cap T = \phi$

(2)#

3.

$60 + 45 + \frac{135}{4} + \cdots + 60 \times \left(\frac{3}{4}\right)^{n-1} \geq 200$

$\Rightarrow \frac{60\left(1-\left(\frac{3}{4}\right)^n\right)}{1-\frac{3}{4}} \geq 200 \Rightarrow 1-\left(\frac{3}{4}\right)^n \geq \frac{50}{60} \Rightarrow \left(\frac{3}{4}\right)^n \leq \frac{1}{6}$

取 log $\Rightarrow \log\left(\frac{3}{4}\right)^n \leq \log\left(\frac{1}{6}\right) \Rightarrow n(\log 3 - \log 4) \leq (-\log 6) \Rightarrow n \geq \frac{-0.7781}{0.4771-0.6020} = 6.\cdots$

$\Rightarrow n \geq 7$ (4)#

4. $B_2 = Ax^2$, $B_3 = Ax^3$, $B_5 = Ax^5$ $\Rightarrow A = \frac{(Ax^2)(Ax^3)}{Ax^5} = \frac{B_2 \times B_3}{B_5} = \frac{982 \times 1056}{1220} \fallingdotseq 850$ (5)#

5.

$1+9+\cdots+9+1 = C^9_0 + C^9_1 + C^9_2 + \cdots + C^9_8 + C^9_9$

$= C^9_0 \cdot 1^0 1^9 + C^9_1 \cdot 1^1 \cdot 1^8 + \cdots + C^9_8 \cdot 1^8 \cdot 1^1 + C^9_9 \cdot 1^9 \cdot 1^0$

$= (1+1)^9 = 2^9 = 512$ (3)#

6. 米蘭最多2晚:

case 1: 米蘭1晚 ⇒ 羅馬 → 翡冷翠 → 威尼斯 → 米蘭 → 羅馬

(1) (2) (3) (1) (1)

還剩6晚 a, b, c, 1, d　　$H^4_6 = C^9_6 = 84$

(0,0,0,6): $\frac{4!}{3!} = 4$　(0,0,2,4): $\frac{4!}{2!} = 12$　(0,1,1,4): 12　(1,1,1,3): 4

(0,0,1,5): $\frac{4!}{2!} = 12$　(0,0,3,3): $\frac{4!}{2!2!} = 6$　(0,1,2,3): 24　(1,1,2,2): 6　共84

(0,2,2,2): 4

case 2: 米蘭2晚 ⇒ (1) (2) (3) (2) (1)

還剩5晚　　$H^4_5 = C^8_5 = 56$

(0,0,0,5): $\frac{4!}{3!} = 4$　(0,1,1,3): $\frac{4!}{2!} = 12$

(0,0,1,4): $\frac{4!}{2!} = 12$　(0,1,2,2): $\frac{4!}{2!} = 12$　共56

(0,0,2,3): $\frac{4!}{2!} = 12$　(1,1,1,2): $\frac{4!}{3!} = 4$

$84 + 56 = 140$ (4)#

7. 實係數多項式 $f(x)=0$ 有一根 i，⇒ 有另一根 $-i$

∴ $f(x)$ 有因式 $(x-i)(x+i)=x^2+1$ ⇒ $f(x)=\underline{(x^2+1)(x)}=x^3+x$

∵ x^3 係數 $=1$；x^2 係數 $=0$

(1) $a=1$ (o)

(2) $b=0$ (x)

(3) $f(x)$ 是奇函數 (o)

(4)

$$\begin{array}{r|rrrr} 1\ -1 & 1 & 0 & 1 & 0 \\ & 1 & -1 & & \\ \hline & & 1 & 1 & \\ & & 1 & -1 & \\ \hline & & & 2 & 0 \\ & & & 2 & -2 \\ \hline & & & & 2 \end{array}$$

商：$1\ 1\ 2$

$f(x)$ 被 $(x-1)$ 除 (x)

得商：x^2+x+2

餘：2

(5) $\begin{cases} y=x^3+x \\ y=x \end{cases}$ ⇒ $x^3=0$ ⇒ $x=0$ ⇒ 僅1個交點 (x)

$\underline{(1)(3)}$ ※

8.

(1) $(\sqrt{2})^{-1}=\frac{1}{\sqrt{2}}>0=\log_{\sqrt{2}}1$ (o)

(2) $(\sqrt{2})^2=2=\log_{\sqrt{2}}2$ (x)

(3) $(\sqrt{2})^4=4=\log_{\sqrt{2}}4$ (o)

(4) $(\sqrt{2})^8=16>6=\log_{\sqrt{2}}8$ (o)

(5) $(\sqrt{2})^{\sqrt{8}}=2^{\sqrt{2}}$ $\xrightarrow{\text{取log}}$ $\log 2^{\sqrt{2}}=\sqrt{2}\times\log 2 \doteqdot 1.414\times 0.3010\doteqdot 0.43$

$\log_{\sqrt{2}}\sqrt{8}=3$ $\quad \log 3=0.4771$ (x)

$\underline{(1)(3)(4)}$ ※

9.

(1) $P(X_1=1)=\dfrac{C^1_1\cdot C^8_4}{C^9_5}=\dfrac{5}{9}$ (x)

(2) $P(X_5=9)=\dfrac{C^1_1 C^8_4}{C^9_5}=\dfrac{5}{9}$ (x)

(3) $X_1=1$ 條件下，剩下8數取4數

$P(X_5=9)=\dfrac{C^1_1 C^7_3}{C^8_4}=\dfrac{4}{8}=\dfrac{1}{2}$ (o)

(4) $P(X_3=5)=\dfrac{C^4_2\cdot C^1_1\cdot C^{n-5}_2}{C^n_5}=\dfrac{6\times\frac{(n-5)(n-6)}{2}}{\frac{n(n-1)(n-2)(n-3)(n-4)}{120}}=\dfrac{360(n-5)(n-6)}{n(n-1)(n-2)(n-3)(n-4)}$ (o)

(5) $P_9=\dfrac{360\times4\times3}{9\times8\times7\times6\times5}=\dfrac{360\times4}{9\times8\times7\times6}\times\dfrac{3}{5}$

$P_{10}=\dfrac{360\times5\times4}{10\times9\times8\times7\times6}=\dfrac{360\times4}{9\times8\times7\times6}\times\dfrac{5}{10}$

∴ $P_9>P_{10}$ (o)

$\underline{(3)(4)(5)}$ ※

10.

(1) $a=3$, $\overrightarrow{AB}=(-4,2)$, $\overrightarrow{AC}=(2,2)$, $\triangle ABC$面積 $=\frac{1}{2}\left|\begin{vmatrix}-4&2\\2&2\end{vmatrix}\right|=6$ (o)

(2) $a=5$, $\overrightarrow{AB}=(-4,0)$, $\overrightarrow{AC}=(2,0)$ $\Rightarrow \overrightarrow{AB}\parallel\overrightarrow{AC}$ $\therefore$ A、B、C共線 (o)

(3) $a=2$,
$\overrightarrow{AB}=(-4,3) \Rightarrow |\overrightarrow{AB}|=5$
$\overrightarrow{AC}=(2,3) \Rightarrow |\overrightarrow{AC}|=\sqrt{13}$
$\overrightarrow{BC}=(6,0) \Rightarrow |\overrightarrow{BC}|=6$
$\therefore |\overrightarrow{BC}|$是最大邊.
$|\overrightarrow{BC}|^2=36<|\overrightarrow{AB}|^2+|\overrightarrow{AC}|^2=38$
$\therefore \triangle ABC$為銳角三角形 (x)

(4) 若 H 是 $\triangle ABC$ 之垂心 $\Rightarrow$
$\overrightarrow{AH}\perp\overrightarrow{BC} \Rightarrow (0,9-a)\perp(6,0) \Rightarrow$ 恆成立
$\overrightarrow{BH}\perp\overrightarrow{AC} \Rightarrow (4,4)\perp(2,5-a) \Rightarrow a=7$
$\overrightarrow{CH}\perp\overrightarrow{AB} \Rightarrow (-2,4)\perp(-4,5-a) \Rightarrow a=7$
$\Rightarrow a=7$ (o)

(5) 若 K 是 $\triangle ABC$ 之外心 $\Rightarrow \overline{KA}=\overline{KB}=\overline{KC} \Rightarrow \sqrt{1^2+(a-6)^2}=\sqrt{3^2+1^2}=\sqrt{3^2+1^2}$
$\therefore$ 當 $a=9$ or 3 時, K 是外心 (x)

(1)(2)(4) #

11.

(1) $\mu_x=\dfrac{76+82+86+80+88+92}{6}=85+\dfrac{(-9)+(-3)+1+(-5)+3+7}{6}=84$ (o)

(2) $\sigma_x=\sqrt{\dfrac{(-8)^2+(-2)^2+2^2+(-4)^2+4^2+8^2}{6}}=\sqrt{\dfrac{168}{6}}>\sqrt{\dfrac{150}{6}}=5$ (x)

(3) 標準化: $\dfrac{x-\mu_x}{\sigma_x} \Rightarrow Z: \dfrac{82-84}{\sigma_x}<0$ (o)

(4) $z=\frac{1}{2}y+50 \Rightarrow r_{xy}=r_{xz}$ (x)

(5) $m=r_{xy}\cdot\dfrac{\sigma_y}{\sigma_x}$
$m'=r_{xz}\cdot\dfrac{\sigma_z}{\sigma_x}=r_{xy}\cdot\dfrac{\frac{1}{2}\sigma_y}{\sigma_x}=\frac{1}{2}m$ (o)

(1)(3)(5) #

12.

(1) $\overrightarrow{AB}\perp\overrightarrow{AC} \Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AC}=0 \Rightarrow (2,-1,3)\cdot(t+1,4-t^2,1)=0$
$\Rightarrow 2t+2-4+t^2+3=0 \Rightarrow t^2+2t+1=0 \Rightarrow t=-1$ (o)

(2) $\overrightarrow{AB}=(2,-1,3)$
$\overrightarrow{AC}=(0,3,1)$
$-10,-2,6$
$\therefore \overrightarrow{AB}\times\overrightarrow{AC}=(-10,-2,6)$ (x)

(3) $\vec{n}\parallel(-10,-2,6)\parallel(5,1,-3) \Rightarrow 5x+y-3z\overset{(-1,0,-1)}{=\!=}-2$ (o)

(4) $\overrightarrow{AO}=(1,0,1)$ $\therefore \cos\theta=\dfrac{(1,0,1)\cdot(-10,-2,6)}{\sqrt{2}\sqrt{140}}=\dfrac{-4}{2\sqrt{70}}=\dfrac{-2}{\sqrt{70}}$ (x)

向量夾角"唯一"

(5) 四面體 OABC 之體積 $=\frac{1}{6}\left|(\overrightarrow{OA}\times\overrightarrow{OB})\cdot\overrightarrow{OC}\right|=\frac{1}{6}\left|(-1,1,1)\cdot(-1,3,0)\right|=\frac{4}{6}$ (o)

$-1,1,1$

(1)(3)(5) #

A. $\sqrt[3]{\underset{\overset{\|}{512\times 3}}{1536}} + \frac{4}{3}\cdot 3^{\frac{2}{6}} + -\underset{\overset{\|}{3^{\frac{1}{3}}\times 3^{-1}}}{\left(3^{\frac{-2}{3}}\right)} = 8\cdot 3^{\frac{1}{3}} + \frac{4}{3}\cdot 3^{\frac{1}{3}} - \frac{1}{3}\times 3^{\frac{1}{3}} = 9\cdot 3^{\frac{1}{3}} = 3^2\cdot 3^{\frac{1}{3}} = 3^{\frac{7}{3}}$ 全模104-4

$\therefore\ \underline{a=\frac{7}{3}}$ ※

B、找規律:

分母 2 $\Rightarrow \overset{1/2}{(2,1)}, \overset{2/2}{(1,2)}$

分母 4 $\Rightarrow \overset{1/4}{(4,1)}, \overset{2/4}{(3,2)}, \overset{3/4}{(2,3)}, \overset{4/4}{(1,4)}$

⋮

分母 16 $\Rightarrow (16,1), (15,2), \cdots$

$\Rightarrow \frac{9}{16}$ 在 $(\underset{\overset{\Uparrow}{後}}{17-9}, \overset{先}{\overset{\Downarrow}{9}}) = \underline{(8,9)}$ ※

C、$L_1 \perp L_2 \Rightarrow \overrightarrow{L_1} \perp \overrightarrow{L_2}$ 且 L_1、L_2 有交点.

① $\overrightarrow{L_1} \perp \overrightarrow{L_2} \Rightarrow (1,2,a)\cdot(3,1,b)=0 \Rightarrow 3+2+ab=0 \Rightarrow ab=-5$

② 有交点 $\begin{cases} x=1+t=-3+3s \cdots (1) \\ y=2+2t=-1+s \cdots (2) \\ z=3+at=-1+bs \end{cases}$ 有解 $\Rightarrow$ 由 (1),(2) 知 $s=1,\ t=-1$.

$\therefore\ 3-a=-1+b \Rightarrow a+b=4$

$\therefore\ a^2+b^2=(a+b)^2-2ab=16+10=\underline{26}$ ※

D. 設 $\vec{a}$ 和 $\vec{a}-\vec{b}$ 夾角 $\theta \Rightarrow \cos\theta = \dfrac{\vec{a}\cdot(\vec{a}-\vec{b})}{|\vec{a}|\,|\vec{a}-\vec{b}|} = \dfrac{|\vec{a}|^2-\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{a}-\vec{b}|}$

$= \dfrac{2\times 2-2\times 4\times\frac{1}{2}}{2\cdot\sqrt{|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2}} = 0 \Rightarrow \theta=\underline{90^\circ}$ ※

E.
$$\begin{array}{cc} & \begin{array}{ccc}甲 & 乙 & (原)\end{array} \\ \begin{array}{c}甲\\乙\\(新)\end{array} & \begin{bmatrix}0.6 & 0.6\\ 0.4 & 0.4\end{bmatrix} \end{array}$$

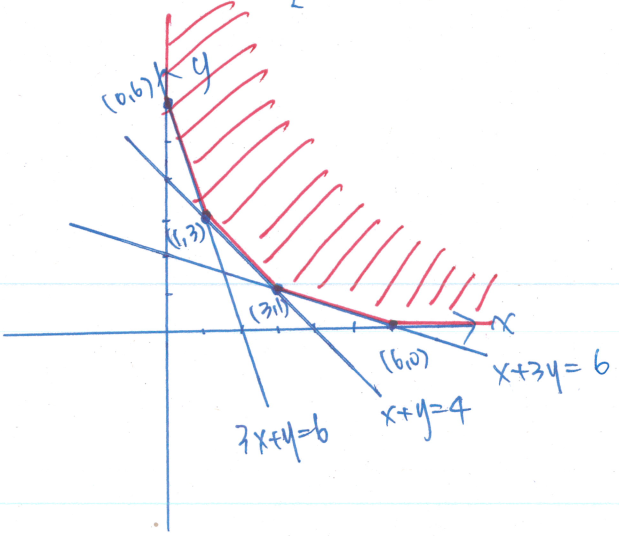
設穩定狀態、甲的市占率 x

乙的市占率 $(1-x)$

$\therefore\ \begin{bmatrix}0.6 & 0.6\\ 0.4 & 0.4\end{bmatrix}\begin{bmatrix}x\\ 1-x\end{bmatrix}=\begin{bmatrix}x\\ 1-x\end{bmatrix} \Rightarrow 0.6x+0.6(1-x)=x$

$\therefore\ x=0.6$ $\underline{60\%}$ ※

F.



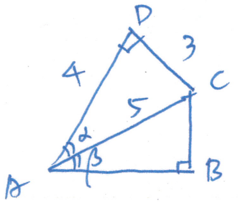
$\begin{cases} x+y=4 \\ 3x+y=6 \end{cases}$ $x=1,\ y=3$

[頂點法]

頂點	(0,6)	(1,3)	(3,1)	(6,0)
$5x+12y$	72	41	27	30

最小值 $=\underline{27}$ ※

9. 由 $\triangle ACD$ 知 $\overline{AC}=5$，設 $\angle CAD=\alpha$，$\Rightarrow \angle CAB=\beta=60^\circ-\alpha$



$\therefore \cos\beta=\cos(60^\circ-\alpha)$

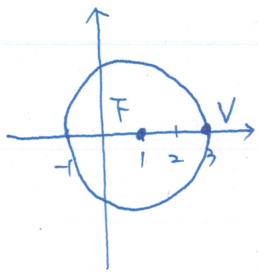
$=\cos60^\circ\cos\alpha+\sin60^\circ\sin\alpha=\frac{1}{2}\times\frac{4}{5}+\frac{\sqrt{3}}{2}\times\frac{3}{5}=\frac{4+3\sqrt{3}}{10}$

$\sin\beta=\sin(60^\circ-\alpha)$

$=\sin60^\circ\cos\alpha-\cos60^\circ\sin\alpha=\frac{\sqrt{3}}{2}\times\frac{4}{5}-\frac{1}{2}\times\frac{3}{5}=\frac{4\sqrt{3}-3}{10}$

$\therefore \overline{BC}+\overline{AB}=5\cdot\sin\beta+5\cdot\sin\alpha=5\cdot\frac{1+7\sqrt{3}}{10}=\frac{1+7\sqrt{3}}{2}$ #

11. $C:(x-1)^2+y^2=4$　　$F(1,0)$.



$P: y^2=ax+b \Rightarrow y^2=a(x+\frac{b}{a})$

$\therefore$ 頂點 $V(\frac{-b}{a},0)$　$\because ab<0$　$\therefore \frac{-b}{a}>0$ 且 頂點在圓周上

$\therefore V(\frac{-b}{a},0)=(3,0)$

$c=F-V=-2$　　$\therefore 4c=a=-8$

$\frac{-b}{-8}=3 \Rightarrow b=24$　　　$(-8,24)$ #