

P3.

$$1. \quad 1) \quad 3^{-\frac{2}{3}} \times (3^{\frac{2}{3}})^{-\frac{1}{2}} \times (3^{\frac{1}{2}})^{\frac{11}{2}} = 3^{-1 - \frac{3}{4} + \frac{11}{4}} = 3^1 = 3$$

$$2) \quad \left(\frac{81}{16}\right)^{-\frac{1}{4}} \times \left(\frac{8}{27}\right)^{-\frac{2}{3}} \times \left(\frac{1}{4}\right)^{-\frac{5}{2}} = \frac{2}{3} \times \frac{9}{4} \times 3^2 = 48$$

$$3) \quad 7^{2+\sqrt{3}} \times (7^3)^{-\frac{\sqrt{3}}{3}} = 7^{2+\sqrt{3}-\sqrt{3}} = 7^2 = 49$$

$$4) \quad \left[ a^{\frac{1}{2}} \times (a \times a^{-\frac{2}{3}})^{\frac{1}{4}} \right]^{\frac{1}{7}} = (a^{\frac{1}{2}} \times a^{\frac{1}{12}})^{\frac{1}{7}} = (a^{\frac{7}{12}})^{\frac{1}{7}} = a^{\frac{1}{12}}, \quad x = \frac{1}{12}$$

$$5) \quad 5^{2.3-0.8-2.5} - \frac{1}{\sqrt{5}} = 5^{-1} - \frac{1}{\sqrt{5}} = \frac{1}{5} - \frac{1}{\sqrt{5}} = \frac{4}{\sqrt{5}}$$

$$6) \quad \left(\frac{25}{9}\right)^{\frac{1}{2}} + \left(\frac{1}{10}\right)^{-1} + 4 \times \left(\frac{64}{27}\right)^{-\frac{1}{3}} + 2^{-3} = \frac{5}{3} + 10 + 4 \times \frac{3}{4} + \frac{1}{8} = \frac{40+240+12+3}{24} = \frac{355}{24}$$

$$7) \quad 2^{10} - 2^2 + \frac{2^{10\sqrt{5}}}{2^{10\sqrt{5}}} = 1024 - 4 + 1 = 1021$$

2.

$$1) \quad 4^x + 4^{-x} = (2^x + 2^{-x})^2 - 2 \cdot 2^x \cdot 2^{-x} = 6^2 - 2 = 34$$

$$2) \quad (2^{\frac{x}{2}} + 2^{-\frac{x}{2}})^2 = 2^x + 2^{-x} + 2 = 6 + 2 = 8, \quad 2^{\frac{x}{2}} + 2^{-\frac{x}{2}} = \sqrt{8} = 2\sqrt{2}$$

$$3) \quad 8^x + 8^{-x} = (2^x + 2^{-x})^3 - 3 \cdot 2^x \cdot 2^{-x} (2^x + 2^{-x}) = 6^3 - 3 \times 1 \times 6 = 198$$

$$3. \quad 1) \quad x^{\frac{3}{2}} + x^{-\frac{3}{2}} = (x^{\frac{1}{2}} + x^{-\frac{1}{2}})^3 - 3 \cdot x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}} (x^{\frac{1}{2}} + x^{-\frac{1}{2}}) = 3^3 - 3 \times 1 \times 3 = 18$$

$$x + x^{-1} = (x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2 - 2 \cdot x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}} = 3^2 - 2 = 7$$

$$x^2 + x^{-2} = (x + x^{-1})^2 - 2 \cdot x \cdot x^{-1} = 7^2 - 2 = 47$$

$$\text{FFK} = \frac{18+7}{47+3} = \frac{25}{50} = \frac{1}{2}$$

$$2) \quad \frac{a^x - a^{-x}}{a^{3x} - a^{-5x}} \cdot \frac{a^x}{a^x} = \frac{a^{2x} - 1}{a^{4x} - a^{-4x}} = \frac{3-1}{3^2 - \frac{1}{3^2}} = \frac{2}{\frac{80}{9}} = \frac{9}{40}$$

$$4. \quad \begin{cases} 2^x = a \Rightarrow 2 = a^{\frac{1}{x}} & \text{--- (1)} \\ 3^y = a \Rightarrow 3 = a^{\frac{1}{y}} & \text{--- (2)} \\ 5^z = a \Rightarrow 5 = a^{\frac{1}{z}} & \text{--- (3)} \end{cases} \quad \text{1.} \cdot \text{2.} \cdot \text{3.} \Rightarrow 30 = a^{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}} = a^2$$

$$\therefore a = \sqrt{30}$$

$$5. \quad \begin{cases} 21^x = 27 = 3^3 \Rightarrow 21 = 3^{\frac{3}{x}} & \text{--- (1)} \\ 189^y = 243 = 3^5 \Rightarrow 189 = 3^{\frac{5}{y}} & \text{--- (2)} \end{cases} \quad \frac{\text{1.}}{\text{2.}} : \frac{1}{9} = 3^{\frac{3}{x} - \frac{5}{y}} = 3^{-2} \Rightarrow \frac{3}{x} - \frac{5}{y} = -2$$

$$6. \quad (1) \quad 450 \times (0.64)^{1.5} = 450 \times \left(\frac{64}{100}\right)^{\frac{3}{2}} = 450 \times \left(\frac{8}{10}\right)^3 = 230.4$$

$$(2) \quad \frac{M(t) - M(t+1)}{M(t)} = \frac{450 \cdot (0.64)^t - 450 \cdot (0.64)^{t+1}}{450 \cdot (0.64)^t} = \frac{1 - 0.64}{1} = 0.36$$

P6.

$$1. \log 100 + \log 10\sqrt{10} - \log \frac{1}{1000} = 2 + \frac{3}{2} - (-3) = \frac{13}{2}$$

$$2. 10^a = 10^{\log 3} = 3, \quad 10^b = 10^{\log 4} = 4$$

$$100^{2a + \frac{1}{2}b} = 10^{4a+b} = (10^a)^4 \times 10^b = 3^4 \times 4 = 324$$

$$3. \log \left( \frac{abc}{100} \right) = \log(abc) - \log 100 = \log a + \log b + \log c - 2 = \frac{1}{2} + 4 + (-2) - 2 = \frac{1}{2}$$

$$4. 3^{-50} = (10^{\log 3})^{-50} = 10^{0.4771 \times (-50)} = 10^{-23.855} = 10^{-24+0.145}$$

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$$5. 47^{100} = (10^{\log 47})^{100} = 10^{100 \log 47} \text{ 是 } 168 \text{ 位數}$$

$$\therefore 100 \log 47 = 167. \dots \Rightarrow \log 47 = 1.67 \dots$$

$$47^{50} = (10^{\log 47})^{50} = 10^{50 \times 1.67 \dots} = 10^{83. \dots} \text{ 故 } \frac{1}{2} \text{ 是 } 84 \text{ 位數}$$

$$6. \text{ (1) } \log 2 - \log 20 = \log \frac{2}{20} = \log \frac{1}{10} = -1$$

$$\begin{aligned} \text{(2) } 3 \frac{\log \sqrt{2}}{\log 2} - \frac{1}{2} \times \frac{\log 3}{\log 2} + \frac{\log \left( \frac{\sqrt{3}}{2} \right)}{\log 2} &= \frac{3 \log \sqrt{2} - \frac{1}{2} \log 3 + \log \frac{\sqrt{3}}{2}}{\log 2} \\ &= \frac{\log (\sqrt{2})^3 - \log \sqrt{3} + \log \frac{\sqrt{3}}{2}}{\log 2} = \frac{\log \frac{2\sqrt{2} \times \sqrt{3}}{\sqrt{3}}}{\log 2} = \frac{\log \sqrt{2}}{\log 2} = \frac{\frac{1}{2} \log 2}{\log 2} = \frac{1}{2} \end{aligned}$$

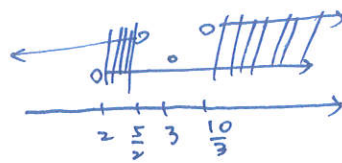
$$\text{(3) } \log \frac{14}{25} - \log 32 - \log 9 + \log \frac{36}{7} = \log \left( \frac{14}{25} \times \frac{1}{32} \times \frac{1}{9} \times \frac{36}{7} \right) = \log \frac{1}{100} = -2$$

$$7. (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$$

$$\text{令 } a = \log 20, \quad b = \log 2, \quad a-b = \log 20 - \log 2 = \log \frac{20}{2} = \log 10 = 1$$

$$\begin{aligned} \text{原式} &= (\log 20)^3 - (\log 2)^3 - (\log 20) \cdot (3 \log 2) - (\log 20)^3 - (\log 2)^3 - 3(\log 20)(\log 2) \frac{(\log 20 - \log 2)}{1} \\ &= (\log 20 - \log 2)^3 = 1^3 = 1 \end{aligned}$$

pg. 1.  $\begin{cases} x-2 > 0 & \Rightarrow x > 2 \\ x-2 \neq 1 & \Rightarrow x \neq 3 \\ 6x^2 - 35x + 50 > 0 \end{cases}$   
 $\hookrightarrow (3x-10)(2x-5) > 0 \Rightarrow x > \frac{10}{3} \text{ or } x < \frac{5}{2}$



$\therefore 2 < x < \frac{5}{2} \text{ or } x > \frac{10}{3}$

2.  $2^x = 2^{\log_2 7} = 7$

$4^x = (2^x)^2 = 7^2 = 49$

$4^x + 2^{-x} = 49 + \frac{1}{7} = \frac{344}{7}$

$2^{-x} = (2^x)^{-1} = 7^{-1} = \frac{1}{7}$

3. (1)  $\log_2 8 = \log_2 2^3 = 3$

(2)  $\log_3 \frac{1}{9} = \log_3 3^{-2} = -2$

(3)  $\log_4 1 = \log_4 4^0 = 0$

(4)  $\log_5 5\sqrt{5} = \log_5 5^{\frac{3}{2}} = \frac{3}{2}$

4. (1)  $\log \frac{2 \times 15}{3} = \log 10 = 1$

(2)  $\log (50 \times \frac{3}{7} \times \frac{14}{3}) = \log 100 = 2$

(3)  $\log \left( \frac{2^3 \times 5^2}{2} \right) = \log 100 = 2$

(4)  $\log (3\sqrt{4}) \times \log 25 = \log (4 \times 25) = \log 100 = 2$

5. (1)  $\log_6 (3 \times 8 \times \frac{3}{2}) = \log_6 6^2 = 2$

(2)  $\log_5 \left( \frac{250 \times 10}{4} \right) = \log_5 5^4 = 4$

(3)  $\log_{18} (2 \times 3^2) = \log_{18} 18 = 1$

(4)  $\log_3 (15 \times \sqrt{30} \times \frac{1}{5\sqrt{5}} \times \frac{1}{\sqrt{2}}) = \log_3 3\sqrt{3} = \log_3 3^{\frac{3}{2}} = \frac{3}{2}$

6. (1)  $(\log_2 7^{\frac{1}{2}}) \times (\log_7 2 + \log_7 2^{-3}) = \left( \frac{1}{2} \log_2 7 \right) \times \left[ \log_7 2 + \left( \frac{-3}{2} \right) \log_7 2 \right]$

$= \frac{1}{2} \log_2 7 \times \left( \frac{-1}{2} \log_7 2 \right) = \frac{-1}{4} \log_2 2 = \frac{-1}{4}$

(2)  $\log_6 3 + \log_6 12 = \log_6 36 = \log_6 6^2 = 2$

15.

$$1. A = \left(\frac{3}{10}\right)^3, B = \left(\frac{3}{10}\right)^{-2}, C = \left(\left(\frac{3}{10}\right)^2\right)^{\frac{5}{2}} = \left(\frac{3}{10}\right)^5, D = \left(\frac{3}{10}\right)^0$$

$$\because -2 < 0 < 3 < 5 \quad \text{且} \quad 0 < \frac{3}{10} < 1$$

$$\therefore \left(\frac{3}{10}\right)^{-2} > \left(\frac{3}{10}\right)^0 > \left(\frac{3}{10}\right)^3 > \left(\frac{3}{10}\right)^5 \quad \text{故} \quad B > D > A > C$$

$$2. (1) 3^{x-1} = (3^2)^{x^2-2}, \quad 3^{x-1} = 3^{2x^2-4}, \quad x-1 = 2x^2-4,$$

$$2x^2 - x - 3 = 0, \quad (2x-3)(x+1) = 0, \quad x = \frac{3}{2} \text{ 或 } -1$$

$$(2) (0.5)^x - (0.5)^{2x+2} = 1, \quad (0.5)^x - (0.5)^2 (0.5)^{2x} - 1 = 0,$$

$$\text{令 } (0.5)^x = t, \Rightarrow t - 0.25t^2 - 1 = 0, \quad 4t - t^2 - 4 = 0,$$

$$t^2 - 4t + 4 = 0 \Rightarrow (t-2)^2 = 0, \quad t = 2, \quad \text{即 } (0.5)^x = 2,$$

$$\left(\frac{1}{2}\right)^x = 2, \quad x = -1$$

$$(3) \left(\frac{1}{10}\right)^x > \left(\frac{1}{10}\right)^{\frac{x}{2}} \quad \because 0 < \frac{1}{10} < 1 \quad \therefore x < \frac{x}{2}, \quad x < 0$$

3.

$$(1) \log(x+10) = \log 10 + \log(x-8), \quad \log(x+10) = \log(10x-80),$$

$$x+10 = 10x-80, \quad 9x = 90, \quad x = 10 \quad (\text{检查真数大于0, 合})$$

$$(2) \frac{\log(x-2)}{\log \sqrt{3}} = \frac{\log(2x-1)}{\log 3}, \quad \frac{\log(x-2)}{\frac{1}{2} \log 3} = \frac{\log(2x-1)}{\log 3}, \quad 2 \log(x-2) = \log(2x-1),$$

$$\log(x-2)^2 = \log(2x-1), \quad (x-2)^2 = (2x-1), \quad x^2 - 4x + 4 = 2x - 1,$$

$$x^2 - 6x + 5 = 0, \quad (x-1)(x-5) = 0, \quad x = 1 \text{ 或 } 5. \quad (\text{检查真数大于0,})$$

$$\Rightarrow x = 5 \quad (x=1 \text{ 不合})$$

$$4. \text{真数大于0} \Rightarrow x-1 > 0, \quad x > 1 \dots \textcircled{1}$$

$$\text{原式 } (x-1) < 2, \quad x < 3 \dots \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \text{ 交集: } \underline{1 < x < 3}$$