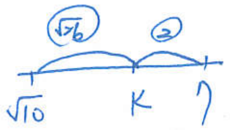
1. 分點公式



$\because \sqrt{26} \approx 5$

選(4)

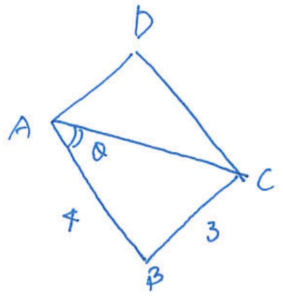
2. $f(x)$: ① 右上 $\Rightarrow a>0$

② 無波峰波谷 $\Rightarrow ap>0$, $p>0$

$g(x) = ax+p$ ① $a>0$: 右上

② $p>0$: y截距 >0 選(3)

3.



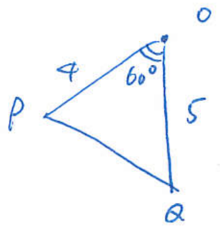
$m_{AB} = -2 = \tan\phi$

$m_{AC} = \tan(\phi-\theta)$ or $\tan(\phi+\theta)$ ，其中 $\tan\theta = \frac{3}{4}$

$= \dfrac{-2-\frac{3}{4}}{1+(-2)(\frac{3}{4})}$ or $\dfrac{-2+\frac{3}{4}}{1-(-2)\times\frac{3}{4}}$

$= \dfrac{-\frac{11}{4}}{-\frac{1}{2}}$ or $\dfrac{\frac{-5}{4}}{\frac{5}{2}} = \frac{11}{2}$ or $-\frac{1}{2}$ 選(2)

4.



$\overline{PQ} = \sqrt{4^2+5^2-2\cdot4\cdot5\cdot\cos60^\circ} = \sqrt{16+25-20} = \sqrt{21}$

$\triangle OPQ$面積 $= \frac{1}{2}\times4\times5\times\sin60^\circ = \frac{1}{2}\times\overline{PQ}\times d(O,\overleftrightarrow{PQ})$

$\Rightarrow 5\sqrt{3} = \frac{1}{2}\times\sqrt{21}\times d(O,\overleftrightarrow{PQ})$

$\Rightarrow d(O,\overleftrightarrow{PQ}) = \frac{10\sqrt{3}}{\sqrt{21}} = \frac{10}{\sqrt{7}}$ ，選(5)

5. $A^5 = I$

$B^2 = I$ $\Rightarrow A^{10} = B^{10}$ ，選(1)

6. (1) $\mu_Y = 8\cdot\dfrac{\mu_X-\mu_X}{\sigma_X}+64 = 64$ (○)

(2) $\sigma_Y = 8\cdot\dfrac{\sigma_X}{\sigma_X} = 8$ (×)

(3) $Y = 8\cdot\dfrac{38-43}{10}+64 = 60$ (○)

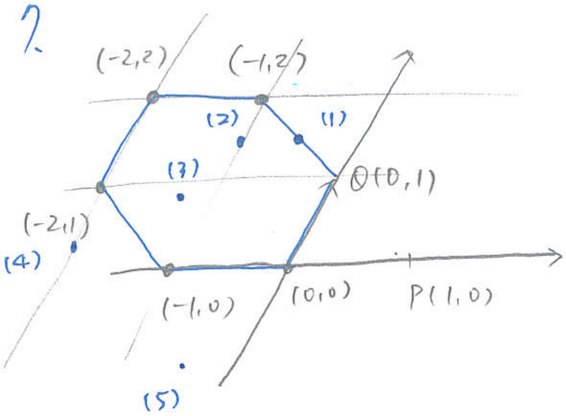
(4) X、Y 關係為斜率為正的直線

$\therefore r_{XY} = 1$ (○)

(5) $\sigma_Y = 8 < \sigma_X = 10$，Y較集中 (○)

選(1)(3)(4)(5)

7.



如圖，選 (2)(3)

8. 最大值 3，最小值 -1 $\Rightarrow$ 基線 $y=1$ $\Rightarrow$ $k=1$

振幅 2 $\Rightarrow$ $a=2$

週期 $= 6 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{3}$

$(0,3)$ 代入得 $3 = 2\sin\left(\frac{\pi}{3}(0+\theta)\right) + 1$，$\sin\frac{\pi}{3}\cdot\theta = 1$

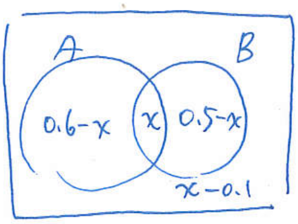
$\frac{\pi}{3}\theta = \frac{\pi}{2} + 2k\pi$，$\theta = \frac{3}{2} + 6k$，其中 $k \in \mathbb{Z}$

若 $f(\alpha)=0$，由週期$=6$知：$\alpha = 2+6k$ or $4+6k$　$k \in \mathbb{Z}$

$\therefore$ α 是偶數．

選 (1)(3)(5)

9.



$0.6-x \geq 0$
$0.5-x \geq 0$
$x \geq 0$
$x-0.1 \geq 0$

$\therefore\ 0.1 \leq x \leq 0.5$

(1) $x \neq 0$．$\therefore$ A、B 不互斥　(○)

(2) 若 A、B 獨立，則 $P(A\cap B) = 0.6\times 0.5 = 0.3$．有可能　(X)

(3) $0.1 \leq x \leq 0.5$　(X)

(4) 會說英文共有 120 人　$\therefore\ P = \frac{C^{120}_2}{C^{200}_2} = \frac{120\times 119}{200\times 199} \neq 0.6\times 0.6$　(X)

(5) $P(法|英) = \frac{x}{0.6}$

$P(英|法) = \frac{x}{0.5}$

$\frac{x}{0.6} < \frac{x}{0.5}$　(○)

選 (1)(5)

10.

(1) $P=\frac{2}{100}=0.02$ (o)

(2) 剩下99個福袋，有1個特獎. $P=\frac{1}{99}$ (o)

(3) $P=\frac{20\times19}{100\times99}\neq\frac{1}{25}$ (x)

(4)

$$\frac{2\times1\times98\times\frac{3!}{2!}}{100\times99\times98}=\frac{3}{4950} \quad (\times)$$

(5) $\frac{2000\times1+1000\times2+500\times17}{100}=20+20+85=125$ (o)

獲利 = 收入 − 支出 $=125-100=25$

選 (1)(2)

11.

(1) $S_n=S_{n-1}+a_n \Rightarrow (2a_{n-1}-1)+a_n=(2a_n-1)$

$\therefore a_n=2a_{n-1}$, $\langle a_n\rangle$ 是公比為 2 的等比數列. (x)

(2) $S_1=2a_1-1 \Rightarrow a_1=2a_1-1 \Rightarrow a_1=1$

$a_n=1\cdot2^{n-1}<10^4\approx(2^{10})\cdot10$

$2^{13}=8192$, $2^{14}>1000$ $\therefore$ 有14項 (o)

(3) $b_1=3$, $b_2=a_1+b_1=4$, $b_3=a_2+b_2=2+4=6$. 非等比 (x)

(4) $b_n=a_{n-1}+b_{n-1}=a_{n-1}+(a_{n-2}+b_{n-2})=\cdots=\underline{a_{n-1}+a_{n-2}+\cdots+a_1}+b_1$

$=S_{n-1}+3=(1+2+\cdots+2^{n-2})+3=2^{n-1}-1+3=2^{n-1}+2$

$a_n-b_n=2^{n-1}-(2^{n-1}+2)=-2$ (o)

(5) 承(4), $\underbrace{b_n=2^{n-1}+2}_{\text{是偶數 } (n\geq2)}$, 但 $b_1=3$. $\therefore$ 奇+偶+偶+偶+⋯ 必為偶數 (o)

選 (2)(4)(5)

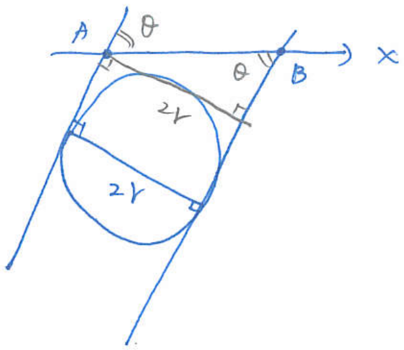
12. "+1" x次, "−1" y次

4次 $\Rightarrow \frac{4!}{4!}=1$ 8次 $\Rightarrow \frac{8!}{6!2!}=28$

6次 $\Rightarrow \frac{6!}{5!1!}=6$ 10次 $\Rightarrow \frac{10!}{7!3!}=120$

共 155 種

13.



$\because m = 2 = \tan\theta$

又 $\dfrac{2r}{\overline{AB}} = \sin\theta$

$\therefore \dfrac{2(2\sqrt{5})}{\overline{AB}} = \dfrac{2}{\sqrt{5}}$ $\therefore \underline{\overline{AB} = 10}$ #

14. $\overrightarrow{OP} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$ 表示 $\overrightarrow{OA}, \overrightarrow{OB}$ 所展成之平面.

$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 2 \end{array}$

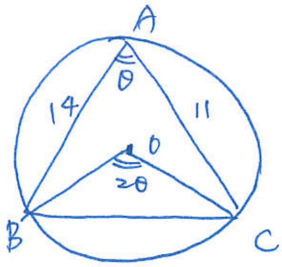
$(-3, 4, -1)$

$\therefore P$ 點所形成之軌跡 $\Rightarrow -3x + 4y - z = 0$ 過 $O(0,0,0)$

$E: 3x - 4y + z = 0$

$\left|\overrightarrow{OP} - \overrightarrow{OC}\right|$ 之最小值 $= d(C, E) = \dfrac{|15 + 12 - 1|}{\sqrt{3^2 + 4^2 + 1^2}} = \dfrac{26}{\sqrt{26}} = \underline{\sqrt{26}}$ #

15.



$\dfrac{\overline{BC}}{\sin\theta} = R_1 \cdots ①$

$\dfrac{\overline{BC}}{\sin 2\theta} = R_2 = 2R_1 \cdots ②$

$\therefore \dfrac{②}{①} : \dfrac{\sin\theta}{\sin 2\theta} = 2$

$\therefore \dfrac{1}{2\cos\theta} = 2, \ \cos\theta = \dfrac{1}{4}$

$\overline{BC} = \sqrt{14^2 + 11^2 - 2 \times 14 \times 11 \times \frac{1}{4}} = \sqrt{196 + 121 - 77} = \sqrt{240}$

$= \underline{4\sqrt{15}}$ #

16. $95 = 10(\log I_{10} + 12) \Rightarrow \log I_{10} = 9.5 - 12 = -2.5, \ I_{10} = 10^{-2.5}$

$\dfrac{I_{10}}{I_{100}} = \left(\dfrac{100}{10}\right)^2 = 100, \ I_{100} = \dfrac{10^{-2.5}}{100} = 10^{-4.5}$

$dB = 10(\log 10^{-4.5} + 12) = 10(-4.5 + 12) = \underline{75}$ #

17. $M\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & -2\sqrt{3} \\ 1 & 2 \end{bmatrix} \Rightarrow (\det M) \times 2 = 4\sqrt{3}, \ \det M = 2\sqrt{3}$

$\triangle PQR$ 面積 $= 3\sqrt{7} \times 2\sqrt{3} = 6\sqrt{21}$. 又 $\overline{PQ} = \sqrt{(3\sqrt{3})^2 + 1^2} = \sqrt{28}$ $\therefore 6\sqrt{21} = \frac{1}{2} \times \sqrt{28} \times d(R, \overleftrightarrow{PQ})$

$d(R, \overleftrightarrow{PQ}) = \dfrac{6\sqrt{21}}{\sqrt{7}} = \underline{6\sqrt{3}}$ #

18、① $\overrightarrow{AE} \parallel n_{EFGH} = (1,2,2)$

② 點 $(3,-5,4)$ ，選(1) #

19. 邊長 $a = d(A, EFGH\text{平面}) = \dfrac{|3-10+8+5|}{\sqrt{1^2+2^2+2^2}} = \dfrac{6}{3} = 2$

② 坐標化(自己架). $H(0,0,0)$, $E(2,0,0)$, $G(0,2,0)$, $D(0,0,2)$

得 $A(2,0,2)$, $P(1,2,1)$

$\triangle APH = \frac{1}{2}\sqrt{|\overrightarrow{HA}|^2|\overrightarrow{HP}|^2-(\overrightarrow{HA}\cdot\overrightarrow{HP})^2} = \frac{1}{2}\sqrt{8\times 6-4^2} = \frac{1}{2}\sqrt{32} = \underline{2\sqrt{2}}$ #

20、承19. 先求平面AHP之方程式

$\begin{matrix} 0 & 2 & 2 & 0 \\ 2 & 1 & 1 & 2 \end{matrix}$: E_{AHP}: $x - z = 0$

$(-4,0,4) \parallel (1,0,-1)$; H(0,0,0)

又 $Q(0,\frac{2}{3},0)$ 代入 E_{AHP} 符合，故 A、H、P、Q 共面 (0)