

1. $\because f(x) = ax^3 + cx \quad \therefore f(0) = 0$ 且 $(0,0)$ 為中心

\because 在 $x=1$ 附近近似 $y = 12(x-1) + 4$

$\therefore \textcircled{1} f(1) = 4 \quad \xrightarrow{(0,0) \text{ 中心}} f(-1) = -4$

$\textcircled{2} f(x) = a(x-1)^3 + b(x-1)^2 + 12(x-1) + 4$

$(0,0)$ 代 $\lambda \Rightarrow 0 = -a + b - 12 + 4$

$(-1, -4)$ 代 $\lambda \Rightarrow -4 = -8a + 4b - 24 + 4$

$$\therefore \begin{cases} a - b = -8 \\ 8a - 4b = -16 \\ 2a - b = -4 \end{cases}$$

$\therefore a = 4, b = 12$

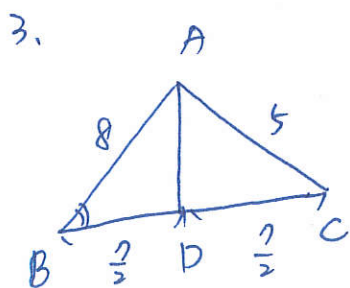
所求 $= f(-1) = 4 \times (-8) + 12 \times 4 + 12(-2) + 4 = -4$ $\xrightarrow{\text{pp (2)}}$

2. 設 $E(0,3)$

$$\frac{\overline{E_1 E}}{\overline{E_1 C}} = \frac{2}{1}, \quad \frac{\overline{E_2 E}}{\overline{E_2 C}} = \frac{2}{3}, \quad \frac{\overline{E_3 E}}{\overline{E_3 C}} = \frac{2}{5}, \dots$$

$$y_{E_1} = 3 \times \frac{1}{3} = 1, \quad y_{E_2} = 3 \times \frac{3}{5} = \frac{9}{5}, \quad y_{E_3} = 3 \times \frac{5}{7} = \frac{15}{7}, \quad y_{E_4} = 3 \times \frac{7}{9} = \frac{7}{3}$$

$$\overline{E_1 E_2} = \frac{4}{5}, \quad \overline{E_2 E_3} = \frac{12}{35}, \quad \overline{E_3 E_4} = \frac{4}{7}, \quad \xrightarrow{\text{pp (5)}}$$



設 $AD = x$

$$\cos B = \frac{(\triangle ABD)}{8^2 + (\frac{7}{2})^2 - x^2} = \frac{(\triangle ABC)}{8^2 + 7^2 - 5^2} = \frac{2 \cdot 8 \cdot \frac{7}{2}}{2 \cdot 8 \cdot 7}$$

$$\Rightarrow 2(64 + \frac{49}{4} - x^2) = 64 + 49 - 25 = 88$$

$$\therefore 64 + \frac{49}{4} - x^2 = 44, \quad x^2 = 20 + \frac{49}{4} = \frac{129}{4}$$

$$\cos \angle BAD = \frac{8^2 + \frac{129}{4} - \frac{49}{4}}{2 \cdot 8 \cdot \sqrt{\frac{129}{4}}} = \frac{84}{16 \sqrt{\frac{129}{4}}} = \frac{21}{2\sqrt{129}}, \quad \tan \angle BAD = \frac{5\sqrt{3}}{21} \quad \xrightarrow{\text{pp (2)}}$$

4. 共有 $1+2+\dots+14 = \frac{14 \times 15}{2} = 105$ 個方格

不相鄰 = $1 - \frac{52}{1575}$

$$P = \frac{\overset{\text{上下相鄰}}{\downarrow} (1+2+\dots+14) + \overset{\text{左右相鄰}}{\downarrow} (1+2+\dots+14)}{105 \times 105 \times \frac{1}{2!}} = \frac{\frac{13 \times 14}{2} \times 2}{105 \times 105 \times \frac{1}{2!}} = \frac{13 \times 2}{15 \times 105 \times \frac{1}{2!}} = \frac{52}{1575}$$

5. ^沒 A 被選的機率 = $(\frac{2}{3})^5$ 有角色沒被選的期望值

B 沒被 ... = $(\frac{2}{3})^5$ = $(\frac{2}{3})^5 \times 3 = \frac{32}{81}$

C = $(\frac{2}{3})^5$ ∴ 角色被選期望值 = $3 - \frac{32}{81} = \frac{211}{81}$ (註(15))

6. 設 $\overline{AB} = x$, $\overline{PO} = y$, 平行線間距 h

$$\overline{HG} = \frac{4x+3y}{7}, \quad \overline{EF} = \frac{5x+2y}{7}$$

$$\triangle ABHG = \frac{(x + \frac{4x+3y}{7}) \times 3h}{2} = \sqrt{2} \Rightarrow \frac{11x+3y}{7} = \frac{2\sqrt{2}}{3h} \dots (1)$$

$$\triangle EFPO = \frac{(\frac{5x+2y}{7} + y) \times 5h}{2} = \sqrt{3} \Rightarrow \frac{5x+9y}{7} = \frac{2\sqrt{3}}{5h} \dots (2)$$

所求 $\triangle ABPO = \frac{(x+y) \times 7h}{2} = \frac{1}{2} \times \frac{1}{3} \times \left(\frac{10\sqrt{2}+12\sqrt{3}}{15h} \right) \times 7h = \frac{35\sqrt{2}+42\sqrt{3}}{45}$ (註(13))

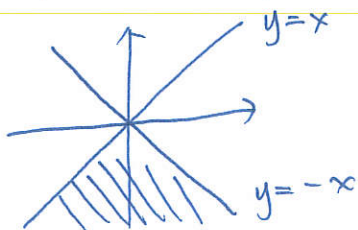
(1), (2) \Rightarrow $(11x+3y) + a(5x+9y)$
 $11+5a = 3+9a, a=2$

(1)+(2) $\times 2$: $\frac{21x+21y}{7} = \frac{2\sqrt{2}}{3h} + \frac{4\sqrt{3}}{5h}$

∴ $x+y = \frac{1}{3} \left(\frac{10\sqrt{2}+12\sqrt{3}}{15h} \right)$

7. $\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}$

∴ $\frac{y}{r} < \frac{x}{r}$ 且 $\frac{y}{r} < \frac{-x}{r} \Rightarrow y < x$ 且 $y < -x$



註(14)(15)

8. (1) 起點、終點均為頂點： $C_1^p = 8$ (0)

e) $64 - 8 = 56$ (x)

(3) $\vec{AG}, \vec{BF}, \vec{CE}$ 及 $\vec{GA}, \vec{FB}, \vec{EC}$ 及 另外 8 個 $\vec{0}$. (x)

(4) $\because \vec{AC} \cdot \vec{AF} < 0$ 且 $\vec{AC} \cdot \vec{FA} > 0$

(5) $\therefore > 0, < 0$ 各半.

承(2), (3) $56 - 6 = 50, \frac{50}{2} = 25$, 得 $\vec{a}_i \cdot \vec{AC} > 0$ 及 $\vec{a}_i \cdot \vec{AC} < 0$ 各 25 (E)

9. $999 \times 30\% = 299.7$, 即 P_{30} 為第 300 筆資料 (由小到大排序)

(1) 若資料有正有負, 則排序無法確定 (x)

(2) 承(1), 無法判定 (x)

(3) 2^x 是遞增, 排序不變 (0)

(4) 承(1), 無法判定 (x)

(5) $\log x$ 是遞增, 排序不變 (0)

EO
選(3)(5) *

10. $\vec{AB} = x \vec{u}$, 其中 $x > 0$

$\vec{AC} = y \vec{v}$, 其中 $y > 0$

$$\vec{BC} = \vec{AC} - \vec{AB} = y\vec{v} - x\vec{u} = (y - 3x, y + x)$$

(1)
$$\begin{cases} y - 3x = 1 \\ y + x = 2 \end{cases}, x = \frac{1}{4}, y = \frac{7}{4} \quad (0)$$

(2)
$$\begin{cases} y - 3x = -1 \\ y + x = 2 \end{cases}, x = \frac{3}{4}, y = \frac{5}{4} \quad (0)$$

(3) $y + x = -2$ (不合)

(5)
$$\begin{cases} y - 3x = -9 \\ y + x = 3 \end{cases}, x = 3, y = 0 \quad (x)$$

EO
選(1)(2) *

11. $A + A^{-1} = I$ $\xrightarrow{\text{同乘 } A}$ $A^2 + I = A$, $A^2 - A + I = 0$

$\therefore (A+I)(A^2-A+I) = 0$, $A^3 + I = 0$, $A^3 = -I$

$a = -1, b = 0, c = 0, d = -1, ad - bc = 1$

選 (2)(4)

12. 1) $\overline{MO} + \overline{EI} = 10$, $\therefore \overline{MO} = 6$ (x)

2) $OS \in p \perp N$, 若要能圍上, $\overline{OS} \leq \overline{JN}$ (x)

3) $SU \in p \perp NP$, $\therefore \overline{SU} \leq \overline{NP}$ (o)

4) $\exists c \perp (4)$, $SU \leq 6$ 公分 $\therefore SU$ 最多 600 個 (o)

5) $\overline{JN} = \overline{OS} = 7$, $PC \leq 3$ 公分 $\therefore PC$ 最多 300 個 (x)

選 (3)(4)

13. 總成本 = $3000 + 30 \times 1000(1 - x\%)$

總收入 = $50(1 + x\%) \times 1000 \times (1 - x\%)$

淨利 = 收入 - 成本 = $1000(1 - x\%) [50(1 + x\%) - 30] - 3000$

$= 1000(1 - \frac{x}{100})(20 + 50\frac{x}{100}) - 3000$

$= 1000(20 + 30t - 50t^2) - 3000$

$\therefore t = \frac{-3}{-10} = 30\%$ 時, 有 $M_{\max} \Rightarrow$ 此時售價 $50(1 + 30\%) = 65$

14. $O(0,0)$, $d(10,4) = \frac{|0+0-2|}{\sqrt{(\frac{1}{4})^2 + (\frac{1}{3})^2}} = \frac{24}{5} = 4.8 \dots$

$r = 5 \sim 10$ 都有 2 個交點, 共 12 個

15. $P(\overline{E} | \overline{M}) = \frac{n(\overline{M} \cap \overline{E})}{n(\overline{M})} = \frac{x}{24} = \frac{2}{3}$, $x = 16$

| | | |
|----|----|----|
| | M | E |
| R | 12 | 16 |
| 不R | 24 | 20 |

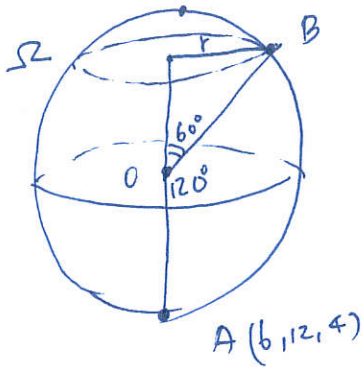
$\therefore n(\overline{M} \cap \overline{E}) = 16$

不R $P(M | \overline{E}) = \frac{n(\overline{E} \cap M)}{n(\overline{E})} = \frac{16}{20} = \frac{4}{5}$

16. $S_9 = \log 1 + \log 2 + \dots + \log 9 = \log 9!$

$= \log 5040 \times 8 \times 9 = \log 3 \dots \times 10^5 = 5 \dots \Rightarrow \underline{5} \#$

17.



設圓 Ω 上 - 點 B

$\widehat{AB} = \frac{2\theta}{3} \pi = 2\pi R \times \frac{\theta}{360^\circ}$, $\frac{\theta}{360^\circ} = \frac{1}{3}$, $\theta = 120^\circ$

\therefore 圓 Ω 半徑 $r = R \cdot \sin 60^\circ = \frac{\sqrt{3}}{2} \times 14 = \underline{7\sqrt{3}} \#$

球 O 半徑

$R = \sqrt{6^2 + 12^2 + 4^2} = 14$

18. 設本金 N

1) $a_n = N(1 + r\% \times n) \Rightarrow a_{n+1} - a_n = N \times r\% \rightarrow$ 是等差

$b_n = N(1 + r\%)^n \Rightarrow \frac{b_{n+1}}{b_n} = 1 + r\% \rightarrow$ 是等比.

例 (3) \rightarrow

19. 單利: 50 年

複利: $\frac{70}{2} = 35$ 年

差 15 年 $\#$

20. 單利: $\frac{200}{2} = 100$ 年

複利: $(1 + 2\%)^n = 3$

$\Rightarrow n \log 1.02 = \log 3$

$\frac{2}{3} (19) \cdot (1 + 2\%)^{35} = 2$

$35 \log 1.02 = \log 2$

$\therefore \frac{n}{35} = \frac{0.4771}{0.3010} \Rightarrow n \approx 55$ 年

差 15 年 $\#$