1、與 x,y 軸相切，則圓心為 $(r,r),(r,-r),(-r,r),(-r,-r)$，其中 r 為半徑

(1) $(x+\frac{1}{2})^2+(y+\frac{1}{2})^2=\frac{3}{2}$ (X)　(2) $(x+\frac{1}{2})^2+(y+\frac{1}{2})^2=1$ (X)

(3) $(x+\frac{1}{2})^2+(y+\frac{1}{2})^2=\frac{3}{4}$ (X)　(4) $(x+\frac{1}{2})^2+(y+\frac{1}{2})^2=\frac{1}{2}$ (X)

(5) $(x+\frac{1}{2})^2+(y+\frac{1}{2})^2=\frac{1}{4}$ (O)

選 (5) #

2. $3.5=2^{t-2}-2^{3-t}$，$\frac{7}{2}=\frac{2^t}{4}-\frac{8}{2^t}$，$14\cdot 2^t=(2^t)^2-32$，

$(2^t)^2-14\cdot 2^t-32=0$，$2^t=16$ or -2 (不合)，$t=4$，選 (2) #

3. $C=B\times\log_2(1+\frac{S}{N})$，$2^{\frac{C}{B}}=1+\frac{S}{N}$，$\frac{S}{N}=2^{\frac{C}{B}}-1$

$$\begin{cases} 999=2^{\frac{C_{999}}{B}}-1 \Rightarrow 2^{\frac{C_{999}}{B}}=1000, & \frac{C_{999}}{B} \\ 1999=2^{\frac{C_{1999}}{B}}-1 \Rightarrow 2^{\frac{C_{1999}}{B}}=2000 \end{cases}$$

$C_{999}=B\times\log_2(1+999)$ ⋯ ①

$C_{1999}=B\times\log_2(1+1999)$ ⋯ ②，$\frac{②}{①}$：$\frac{C_{1999}}{C_{999}}=\frac{\log_2 2000}{\log_2 1000}=\frac{\log 2000}{\log 1000}$

$=\frac{\log 2+3}{3}\doteq\frac{3.3}{3}=1.1$

意即增加 10%，選 (1) #

4. 反向噪音 $y\to -y$

∴ 反向噪音波 $y=-\sin(2x-\frac{\pi}{3})-1$

$=\sin(2x-\frac{\pi}{3}+\pi)-1=\sin(2x+\frac{2\pi}{3})-1$

$\sin(180^\circ+\theta)=-\sin\theta$　$=\cos(\frac{\pi}{2}-(2x+\frac{2\pi}{3}))-1=\cos(-2x-\frac{\pi}{6})-1$

$=\cos(2x+\frac{\pi}{6})-1$

$\cos(-\theta)=\cos\theta$

選 (5) #

5. $a_1>0$, $a_2<0$, $a_3>0$, ... 所以 $\begin{cases} \Pi_n>0, & n=4k+1 \text{ or } 4k \\ \Pi_n<0, & n=4k+2 \text{ or } 4k+3 \end{cases}$ ...①

又 $a_n=2023\times(-\frac{1}{2})^{n-1}$, 故當 $n\le 11$ 時, $|a_n|>1$ ...②

由①、②知, $\Pi_{12}>\Pi_{13}>0>\Pi_{11}$ 且 $\Pi_9>\Pi_8>0$

$\Pi_{12}=\Pi_9\times a_{10}\times a_{11}\times a_{12}$

$|a_{11}|>1$, $|a_{10}|>2$, $|a_{12}|>\frac{1}{2}$

故 $|a_{11}\times a_{10}\times a_{12}|>1$ $\therefore \Pi_{12}>\Pi_9$, 選(4)

6.

$AB=\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}p&q\\r&s\end{bmatrix}=\begin{bmatrix}p+2r&q+2s\\3p+4r&3q+4s\end{bmatrix}$ (1,1)元知: $2r=3q$

$BA=\begin{bmatrix}p&q\\r&s\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}p+3q&2p+4q\\r+3s&2r+4s\end{bmatrix}$

得 $\underbrace{r=0,\ q=0}_{\text{case1}}$, $\underbrace{r=9,\ q=6}_{\text{case 2}}$

case1: $\begin{bmatrix}p&2s\\3p&4s\end{bmatrix}=\begin{bmatrix}p&2p\\3s&4s\end{bmatrix}$

得 $p=s$

case 2:

$\begin{bmatrix}p+18&6+2s\\3p+36&18+4s\end{bmatrix}=\begin{bmatrix}p+18&2p+24\\9+3s&18+4s\end{bmatrix}$

$\begin{cases}6+2s=2p+24\\3p+36=9+3s\end{cases}$, $s-p=9$

$\therefore \underline{p=0,\ s=9}$ or $\underline{p=3,\ s=12}$

case1 有 5 種

case2 有 2 種, 共 7 種, 選(3)

7.

(1) 正弦定理：$\frac{a}{\sin A} = \frac{b}{\sin B}$ (X)

(2) $\because \angle A > \angle B \Rightarrow a > b$ (大角對大邊) $\therefore \sin A > \sin B$

$\therefore a \cdot \sin A > b \cdot \sin B$ 又 $a, b > 0$ $\therefore \frac{\sin A}{b} > \frac{\sin B}{a}$ (O)

(3) 由(2)知，$\sin A > \sin B$ (O)

(4) $\because \angle A > \angle B$ $\therefore \cos A < \cos B$ (X)

(5) 若 A 為鈍角，則 $\tan A > \tan B$

若 B 為鈍角，則 $180° - A$ 為鈍角

又 $A + B + C = 180°$，$180° - A = B + C > B$

$\therefore \underbrace{\tan(180° - A)}_{= -\tan A} > \tan B$，得 $|\tan A| > |\tan B|$ (O)

選 (2)(3)(5)

8.

採方法一的期望值 $E(X_1) = n$

採方法二的期望值 $E(X_2) = 1 \cdot (1-p)^n + (n+1) \cdot [1 - (1-p)^n]$

$= (n+1) - n \cdot (1-p)^n$

X_2	全陰性	至少一人陽性
P	$(1-p)^n$	$1-(1-p)^n$
次數	1	$n+1$

$E(X_1) > E(X_2)$

$\Rightarrow n > (n+1) - n(1-p)^n$，$n(1-p)^n > 1$

由選項知：(1)(2)(3) 考慮 $p = \frac{1}{4} \Rightarrow n \cdot (\frac{3}{4})^n > 1$，$(\frac{3}{4})^n > \frac{1}{n}$

$n \log \frac{3}{4} > -\log n$，$n(0.4771 - 0.6020) > -\log n$

$\frac{\log n}{n} > 0.1249$

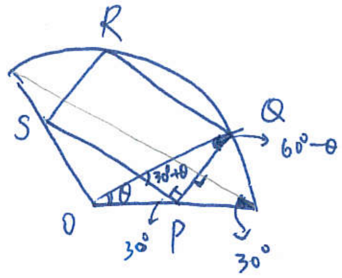
$\frac{\log 5}{5} = 0.1398$ (O) $\frac{\log 6}{6} = 0.1297$ (O) $\frac{\log 7}{7} = 0.1207$ (X)

(4)(5) $p=\frac{1}{5}$. $\Rightarrow$ $n\left(\frac{4}{5}\right)^n>1$, $\left(\frac{8}{10}\right)^n>\frac{1}{n}$

$n(0.9030-1)>-\log n$, $\frac{\log n}{n}>0.097$

$\frac{\log 10}{10}=0.01$ (0) $\frac{\log 11}{11}=0.0947$ (×) 選(1)(2)(4)

9.



(1) $\overline{RQ}=\overline{PS}=\sqrt{3}\,\overline{OP}$ ($30^\circ-30^\circ-120^\circ$, $\triangle OPS$)

$$\frac{\overline{OP}}{\sin(60^\circ-\theta)}=\frac{\overline{OQ}^{\,=r=1}}{\sin(120^\circ)} \Rightarrow \overline{OP}=\frac{2}{\sqrt{3}}\sin(60^\circ-\theta)$$

$\therefore\ \overline{RQ}=2\sin(60^\circ-\theta)$ (0)

(2) $\angle OPQ=120^\circ$ (0)

(3) $\frac{\overline{PQ}}{\sin\theta}=\frac{\overline{OQ}}{\sin 120^\circ}$, $\overline{PQ}=\frac{2}{\sqrt{3}}\cdot 1\cdot\sin\theta$ (×)

(4)

$$\overline{PQ}=\overline{PS} \Rightarrow \frac{2}{\sqrt{3}}\sin\theta=2\sin(60^\circ-\theta)=2(\sin60^\circ\cos\theta-\cos60^\circ\sin\theta)$$

$$\therefore\ \frac{2}{\sqrt{3}}\sin\theta=\sqrt{3}\cos\theta-\sin\theta,\quad \xrightarrow{\text{除以}\cos\theta} \frac{2}{\sqrt{3}}\tan\theta=\sqrt{3}-\tan\theta$$

$$\Rightarrow 2\tan\theta=3-\sqrt{3}\tan\theta,\quad (2+\sqrt{3})\tan\theta=3$$

$$\Rightarrow \tan\theta=\frac{3}{2+\sqrt{3}}=3(2-\sqrt{3})=6-3\sqrt{3}\quad (0)$$

(5)

$$\text{PQRS面積}=2\sin(60^\circ-\theta)\cdot\frac{2}{\sqrt{3}}\sin\theta=\frac{4}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)\cdot\sin\theta$$

$$=\frac{2}{\sqrt{3}}\cdot\left(\sqrt{3}\sin\theta\cdot\cos\theta-\sin^2\theta\right)=\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\sin2\theta-\frac{1-\cos2\theta}{2}\right)$$

$$=\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\sin2\theta+\frac{1}{2}\cos2\theta-\frac{1}{2}\right)$$

$$\therefore\ \text{Max}=\frac{2}{\sqrt{3}}\left(\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2}-\frac{1}{2}\right)=\frac{2}{\sqrt{3}}\times\frac{1}{2}=\frac{\sqrt{3}}{3}\ (0)$$

選(1)(2)(4)(5)

10、 中心$(-1,-2)$，設 $f(x)=a(x+1)^3+p(x+1)-2=(x+1)^2[a(x+1)]+p(x+1)-2$

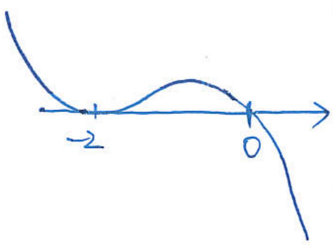
(1) $f(x)$ 除以 $(x+1)^2$ 餘式為 $p(x+1)-2$，不可能為 $x+1$ (○)

(2) $f(x)$ 除以 $(x+1)^2$ 商式為 $a(x+1)$，取 $a=1$ 時，商式為 $(x+1)$ (×)

(3) 在 $x=-1$ 近似直線 $y=p(x+1)-2$，取 $p=3$，近似直線 $y=3x+1$ (×)

(4) 若過點 $(1,-2)$，則 $-2=8a+2p-2$，取 $a=1$，$p=-4$ 即可 (×)

(5)

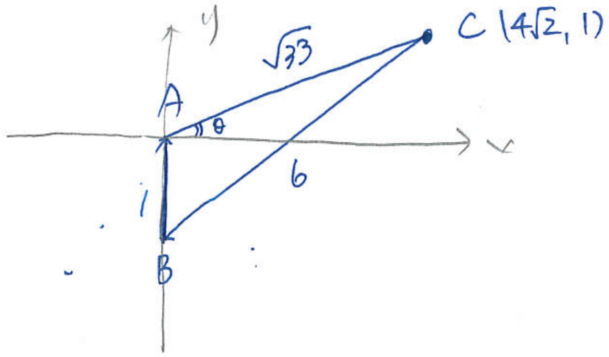


$f(x)<0$ 解為 $x<0$ 且 $x\neq-2$ 的圖形如左。

此時對稱中心 (h,k)，$k>0$ (○)

選(1)(5)

11、 ⑪ 坐標化，$A(0,0)$，$B(0,-1)$，$E(e_1,e_2)$，$F(f_1,f_2)$



$\cos\angle CAB=\cos(90^\circ+\theta)=\dfrac{(\sqrt{33})^2+1^2-6^2}{2\cdot\sqrt{33}\cdot 1}=\dfrac{-1}{\sqrt{33}}$

$\therefore C(\overline{AC}\cos\theta,\ \overline{AC}\cdot\sin\theta)=(4\sqrt{2},1)$

$\overrightarrow{AB}\cdot\overrightarrow{AE}+\overrightarrow{AC}\cdot\overrightarrow{AF}=2 \Rightarrow -e_2+4\sqrt{2}f_1+f_2=2 \cdots ①$

又 $\left(\dfrac{e_1+f_1}{2},\dfrac{e_2+f_2}{2}\right)=(0,-1)$ $\therefore e_1=-f_1,\ e_2=-2-f_2 \cdots ②$

②代入①：$2+f_2+4\sqrt{2}f_1+f_2=2$，$f_2=-2\sqrt{2}f_1$

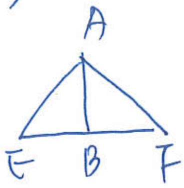
又 $\overline{BF}=\frac{1}{2}$，$\sqrt{f_1^2+(-2\sqrt{2}f_1+1)^2}=\frac{1}{2}$，$f_1^2+8f_1^2-4\sqrt{2}f_1+1=\frac{1}{4}$

$9f_1^2-4\sqrt{2}f_1+\frac{3}{4}=0$，$36f_1^2-16\sqrt{2}f_1+3=0$

$f_1=\dfrac{16\sqrt{2}\pm4\sqrt{5}}{72}=\dfrac{4\sqrt{2}\pm\sqrt{5}}{18}$ (有2解) $\rightarrow$ 麻煩.

11、〔试二〕

(1)



$\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AE} + \frac{1}{2}\overrightarrow{AF}$

$\therefore\ \overrightarrow{AE} = 2\overrightarrow{AB} - \overrightarrow{AF}$ (X)

(2) $\overrightarrow{AB}\cdot\overrightarrow{AC} = |\overrightarrow{AB}|\cdot|\overrightarrow{AC}|\cdot\cos\angle BAC = 1\times\sqrt{33}\times\dfrac{(\sqrt{33})^2+1^2-6^2}{2\times1\times\sqrt{33}} = \dfrac{-2}{2} = -1$ (O)

(3) $\overrightarrow{AE}\cdot\overrightarrow{AF} = (\overrightarrow{AB}+\overrightarrow{BE})\cdot(\overrightarrow{AB}+\overrightarrow{BF}) = |\overrightarrow{AB}|^2 + \overrightarrow{AB}\cdot\overrightarrow{BE} + \overrightarrow{AB}\cdot\overrightarrow{BF} + \overrightarrow{BE}\cdot\overrightarrow{BF}$

$= 1 + \overrightarrow{AB}\cdot\overrightarrow{BE} + \overrightarrow{AB}\cdot\overrightarrow{BF} + \frac{1}{2}\times\frac{1}{2}\times\cos 180^\circ$

$= \frac{3}{4} + \overrightarrow{AB}\cdot(\overrightarrow{BE} + \underbrace{\overrightarrow{BF}}_{= -\overrightarrow{BE}}) = \frac{3}{4}$ (X)

(4) $\overrightarrow{EF}\cdot\overrightarrow{BC} = (\overrightarrow{EA}+\overrightarrow{AF})\cdot(\overrightarrow{BA}+\overrightarrow{AC})$

$= \overrightarrow{EA}\cdot\overrightarrow{BA} + \overrightarrow{EA}\cdot\overrightarrow{AC} + \overrightarrow{AF}\cdot\overrightarrow{BA} + \underline{\overrightarrow{AF}\cdot\overrightarrow{AC}}$ (題目)

$= 2 \quad - \overrightarrow{AE}\cdot\overrightarrow{AC} - \overrightarrow{AB}\cdot\overrightarrow{AF}$

$= 2 - \underbrace{(2\overrightarrow{AB}-\overrightarrow{AF})}_{\text{By (1)}}\cdot\overrightarrow{AC} - \overrightarrow{AB}\cdot\underbrace{(2\overrightarrow{AB}-\overrightarrow{AE})}_{\text{同(1)概念}}$

$= 2 - 2\overrightarrow{AB}\cdot\overrightarrow{AC} + \underline{\overrightarrow{AF}\cdot\overrightarrow{AC}} - 2|\overrightarrow{AB}|^2 + \underline{\overrightarrow{AB}\cdot\overrightarrow{AE}}$ (題目)

$= 2+2-2\overrightarrow{AB}\cdot\overrightarrow{AC} - 2|\overrightarrow{AB}|^2$

$= 4 - 2(-1) - 2\times1^2 = 4$ (O)

(5)

$\triangle$面積 $= \frac{1}{2}\sqrt{|\overrightarrow{EF}|^2|\overrightarrow{BC}|^2 - (\overrightarrow{EF}\cdot\overrightarrow{BC})^2}$

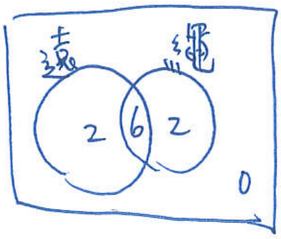
$= \frac{1}{2}\sqrt{1\times36 - 4^2} = \frac{1}{2}\sqrt{20} = \sqrt{5}$ (X)

選 (2)(4)#

12、

(1) 戊的加秒跳繩最差也是並列第8名，必進入決賽 (O)

(4)



由左圖知，壬、癸兩人必定進入跳繩決賽 (O)

(2) 若 $x=60$, $y=59$, 則，乙、丁並列第8名（最差），同時進入決賽，

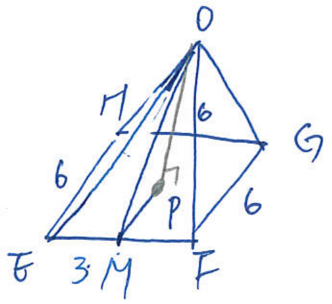
此時，淘汰兩人必為辛、壬，與 (4) 結論矛盾 (×)

(3) 若 $x=59$, $y=58$, $z=60$, 則符合題意，但乙沒進入決賽 (×)

(5) 若 $x=50$, $y=49$, $z=51$, 則符合題意，但 $z<60$ (×)

選 (1)(4)

13.

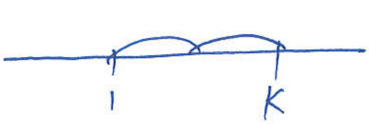


$\overline{OM}=5$, $\overline{PM}=3$

$\therefore \overline{OP}=高=4$

體積 $=\frac{1}{3}\times(6^2)\times 4 = 48$

14.



$|x-1|+|x-k|$ 最小值發生在

x 介於 1 和 k 之間，此值為 $|k-1|\leq 2$

故 $-1\leq k\leq 3$

15、 木棍數　　方法數

$1+5$ ： 1×2

$2+4$ ： 2×2

$3+3$ ： 2×2

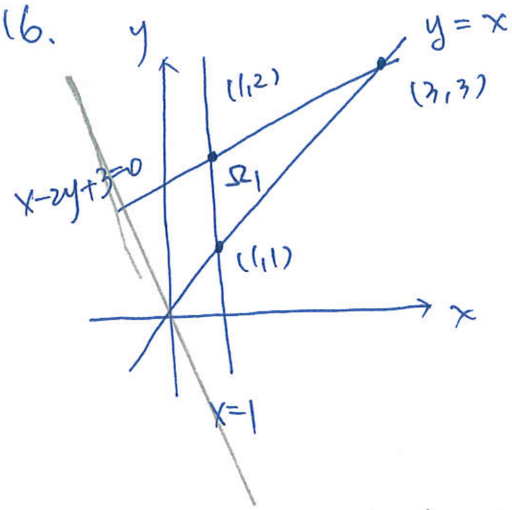
$4+2$ ： 2×2

$5+1$ ： 2×1　　僅有"一"(橫)　　共16種 #

4

有"1"(直) or 沒有"1"(直)

16.



m=-2 (鏡子)

L: $y=-2x$

$M=\begin{bmatrix} \frac{-3}{5} & \frac{-4}{5} \\ \frac{-4}{5} & \frac{3}{5} \end{bmatrix}$ 是鏡射矩陣

其中 $\cos 2\theta = \frac{-3}{5}$， $\sin 2\theta = \frac{-4}{5}$

即 $\tan 2\theta = \frac{4}{3}$， $\frac{2\tan\theta}{1-\tan^2\theta} = \frac{4}{3}$

$3\tan\theta = 2-2\tan^2\theta$， $2\tan^2\theta+3\tan\theta-2=0$

$(2\tan\theta-1)(\tan\theta+2)=0$， $\tan\theta = \frac{1}{2}$ or -2

(不合)

$\because \cos 2\theta < 0$

由圖知A(1,1)距離 $y=-2x$ (鏡子)最近

$\therefore \overline{AB}$ 最小值 $= 2\times d(A,L) = 2\times\frac{|2+1|}{\sqrt{2^2+1^2}} = \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$ #

17.　前3球

O O X

O X O

X O O

P(前3球進2球) $= \frac{1}{2}\times\frac{4}{5}\times\frac{1}{5} + \frac{1}{2}\times\frac{1}{5}\times\frac{2}{3} + \frac{1}{2}\times\frac{2}{3}\times\frac{4}{5}$

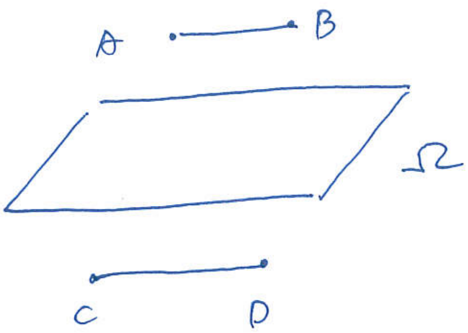
$= \frac{12+10+40}{150} = \frac{62}{150}$

P(第四球進且前3球進2球)

$= \frac{1}{2}\times\frac{4}{5}\times\frac{1}{5}\times\frac{2}{3} + \frac{1}{2}\times\frac{1}{5}\times\frac{2}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{4}{5}\times\frac{4}{5} = \frac{8+8+32}{150} = \frac{48}{150}$

$$\text{所求} = \frac{\frac{48}{150}}{\frac{62}{150}} = \frac{48}{62} = \frac{24}{31} \ \#$$

18、



(1) $\overrightarrow{AB} = (0,0,-15) \parallel (0,0,1)$

$\overrightarrow{CD} = (-8,6,0) \parallel (4,-3,0)$

$\because \overrightarrow{AB} \nparallel \overrightarrow{CD}$ 且 $\overleftrightarrow{AB}$、$\overleftrightarrow{CD}$ 沒有交點

$\therefore \overleftrightarrow{AB}$、$\overleftrightarrow{CD}$ 為歪斜線.

故不共平面 (O)

(2) $\overrightarrow{AB} \cdot \overrightarrow{CD} = 0$ (O)

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \\ 4 & -3 & 0 & 4 & -3 \\ \hline \end{array}$$
$(3, 4, 0)$

(4) Ω 的法向量 $\vec{n} \parallel \overrightarrow{AB} \times \overrightarrow{CD} \parallel (3,4,-3)$

方程式為 $3x+4y=k$

$\because d(A,\Omega) = \dfrac{|0+0-k|}{\sqrt{3^2+4^2}} = 3$, $|k|=15$, $k=\pm 15$

A 代入 $3x+4y=0<15$ ($\because$ A、C 異側 $\therefore k=15$

C 代入 $3x+4y=25>15$ 若 $k=-15$, $0>-15$, $25>-15$, 同側)

$\therefore \Omega: 3x+4y=15$ (O)

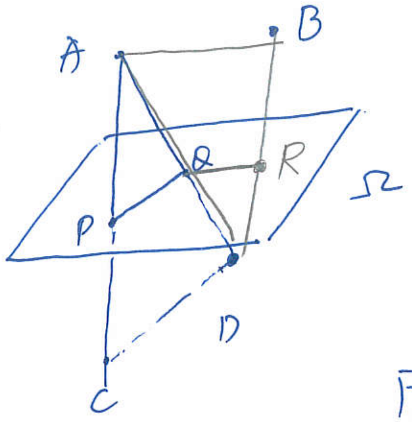
(3) $d(C,\Omega) = \dfrac{|21+4-15|}{\sqrt{3^2+4^2}} = 2$ (×)

(5) ABCD 體積 $= \dfrac{1}{6}\left|\begin{vmatrix} \overrightarrow{AB} \\ \overrightarrow{AC} \\ \overrightarrow{AD} \end{vmatrix}\right| = \dfrac{1}{6} \times \left|\begin{vmatrix} 0 & 0 & -15 \\ 7 & 1 & -12 \\ -1 & 7 & -12 \end{vmatrix}\right| = \dfrac{1}{6} \times \left| -15 \times \begin{vmatrix} 7 & 1 \\ -1 & 7 \end{vmatrix} \right|$

$= \dfrac{1}{6} \times 15 \times 50 = 125$ (×)

選 (1)(2)(4)

19.



∵ $\overline{CD} \parallel \Omega$ ∴ $\overline{PQ} : \overline{CD} = \overline{AP} : \overline{AC}$

$= d(A,\Omega) : d(A,\Omega) + d(C,\Omega)$

∴ $\overline{PQ} : 10 = 3 : 3+2$ ∴ $\overline{PQ} = 6$

同理可得 $\overline{SR} = 6$

同理 $\overline{AB} \parallel \Omega$ ∴ $\overline{QR} : \overline{AB} = d(D,\Omega) : d(D,\Omega) + d(B,R)$

∴ $\overline{QR} : 15 = 2 : 2+3$ ∴ $\overline{QR} = 6$

同理 $\overline{PS} = 6$

四邊長均為6，且 $\overrightarrow{AB} \perp \overrightarrow{CD}$ ∴ $\overrightarrow{PQ} \perp \overrightarrow{QR}$

得 PQRS 面積 $= 6^2 = 36$ #