1.

$a=(3^3-2)^6=25^6=(25^3)^2$

$b=(2\times3^2+3\times3)^4=27^4=(27^2)^2$

$c=(3\times3^2-11)^3=15^3$

$\therefore a>b>c$ 選(5)#

2.

$a\times2^{30}=10^{\log a}\times(10^{\log 2})^{30}=10^{\log a+30\log 2}$

$=10^{\underset{}{30+\underbrace{\log 3}_{0.4771}+\underbrace{30\log 2}_{9.03}}}\approx10^{39.5071}=10^{0.5071}\times10^{39}$

故此數為40位數，選(3)#

3.

$a_n=S_n-S_{n-1}=2n^2-17n-[2(n-1)^2-17(n-1)]=4n-2-17$

$=4n-19$

$b_1\times b_2\times\cdots\times b_K=2^{a_1}\times2^{a_2}\times\cdots\times2^{a_K}=2^{a_1+a_2+\cdots+a_K}>1$，即 $a_1+a_2+\cdots+a_K>0$

$a_5=1$ 是第1個正數

$\underbrace{a_1,a_2,a_3,a_4}$, $a_5$, $\underbrace{a_6,a_7,a_8,a_9}$，故 $K=9$，選(4)#

4.

$m_{AD}=\frac{3}{6}=\frac{1}{2}$，$\overleftrightarrow{AD}$: $y-3=\frac{1}{2}(x+1)$

$m_{BE}=\frac{9}{9}=1$，$\overleftrightarrow{BE}$: $y-5=1\cdot(x-0)$

$\begin{cases}y-3=\frac{1}{2}(x+1)\cdots① \\ y-5=x\cdots②\end{cases}$ ②代入①，$(y-3)\times2=y-5+1$

$2y-6=y-4$，$y=2$，$x=-3$

$\alpha+\beta=-3+2=-1$，選(2)#

5.

$(2021, 3000)$ 代入 $\Rightarrow 3000 = a - b \cdot \log_2 2$

$(2023, 2000)$ 代入 $\Rightarrow 2000 = a - b \cdot \log_2 4$

$\therefore \begin{cases} a-b = 3000 \\ a-2b = 2000 \end{cases}$ , $b = 1000$ , $a = 4000$

所求 $\Rightarrow 4000 - 1000 \cdot \log_2 (x-2019) < 1500$

$1000 \log_2 (x-2019) > 2500$ , $x - 2019 > 2^{2.5} = \sqrt{32} \approx 5.\cdots$

$\therefore x \geq 2025$ , 選 (2) #

6.

平面 ABC 之方程式 : $\frac{x}{4} + \frac{y}{4} + \frac{z}{8} = 1$ , $2x+2y+z = 8$ . $\overrightarrow{n_{ABC}} = (2,2,1)$

平面 OAB 之 $\overrightarrow{n_{OAB}} \parallel \overrightarrow{OA} \times \overrightarrow{OB} \parallel (0,-1,1)$

$\begin{matrix} \cancel{4} & 0 & 0 & 1 & 0 & \cancel{8} \\ \cancel{0} & 1 & 0 & 0 & 1 & \cancel{0} \end{matrix}$

$(0,-1,1)$

$\cos\theta = \left| \frac{(2,2,1)\cdot(0,-1,1)}{3 \times \sqrt{2}} \right| = \left| \frac{-1}{3\sqrt{2}} \right| = \frac{1}{3\sqrt{2}} < \frac{1}{2} = \cos 60^\circ$

$\therefore \theta > 60^\circ$ , 選 (5) #

7.

分人 分房

住 $(2,2,2)$ 人 : $C^6_2 C^4_2 C^2_2 \times \frac{1}{3!} \times 3! = 90$

$(3,2,1)$ 人 : $C^6_3 C^3_2 C^1_1 \times 1 \times 2! = 120$

$\downarrow$

A

共 210 種 , 選 (1) #

8、

(1) $P=\dfrac{50+40}{50+40+70+80}=\dfrac{9}{24}=\dfrac{3}{8}$ (o)

(2) $P=\dfrac{40}{50+40}=\dfrac{4}{9}$ (x)

(3) $P=\dfrac{70}{50+70}=\dfrac{7}{12}$ (x)

(4) 高一女生 $= A\cap B'$

$P(A\cap B')=P(A)\times P(B')=\dfrac{2}{5}\times\dfrac{2}{3}=\dfrac{4}{15}$ (x)

(5) 高二女生 $= A'\cap B' \Rightarrow P(A'\cap B')=P(A')\times P(B')=\dfrac{3}{5}\times\dfrac{2}{3}=\dfrac{2}{5}$

所求 $P(A'\cup B')=P(A')+P(B')-P(A'\cap B')$

$=\dfrac{3}{5}+\dfrac{2}{3}-\dfrac{2}{5}=\dfrac{9+10-6}{15}=\dfrac{13}{15}$ (o)  選 (1)(5)

9、

$\mu_x=\dfrac{500}{100}=5$, $\mu_y=\dfrac{300}{100}=3$

$\sigma_x=\sqrt{\dfrac{3400}{100}-5^2}=3$, $\sigma_y=\sqrt{\dfrac{2500}{100}-3^2}=4$

$r=\dfrac{k-100\times5\times3}{100\times3\times4}=\dfrac{1}{2}$, $k-1500=600$, $k=2100$

$L$: $y-3=\dfrac{1}{2}\cdot\dfrac{4}{3}(x-5)$, $y-3=\dfrac{2}{3}x-\dfrac{10}{3}$

$\therefore y=\dfrac{2}{3}x-\dfrac{1}{3}$, 故 $a=\dfrac{2}{3}$, $b=-\dfrac{1}{3}$

(1) $5a+b=\dfrac{9}{3}=3$ (o)

(2) $\sigma_x=3<4=\sigma_y$ (x)

(3) $k=2100>2000$ (o)

(4) $a=\dfrac{2}{3}>\dfrac{1}{2}$ (o)

(5) $b=-\dfrac{1}{3}>-\dfrac{1}{2}$ (x)  選 (1)(3)(4)

10、

(1) $P=\dfrac{C^2_1C^2_1}{C^{12}_2}=\dfrac{4}{66}=\dfrac{2}{33}$ (x)

(2) 全 $-$ 取同號　6個(同1, 同2, …, 同6)

$P=1-\dfrac{C^2_2+C^2_2+\cdots+C^2_2}{C^{12}_2}=1-\dfrac{6}{66}=\dfrac{60}{66}=\dfrac{10}{11}$ (o)

(3) 和小於6：$\boxed{(1,1),(2,2)}$　$\boxed{(1,2),(1,3),(1,4),(2,3)}$

$P=\dfrac{C^2_2\times 2+C^2_1C^2_1\times 4}{C^{12}_2}=\dfrac{2+16}{66}=\dfrac{9}{33}=\dfrac{3}{11}$ (x)

(4) 承(2),(3)

$P=\dfrac{16}{60}=\dfrac{4}{15}$ (o)

(5) $E$(取2球) $=2\times E$(取1球) $=2\times\dfrac{1}{6}(1+2+3+4+5+6)$

$=2\times\dfrac{21}{6}=\dfrac{21}{3}<8$ (x)

選(2)(4)#

11、

最高點 2公尺
最低點 1公尺　$\Rightarrow$ 中線 1.5公尺，$d=1.5$，$a=0.5$

轉1圈 $15\times 4=60$ 秒，週期 $=60$ $\Rightarrow$ $\dfrac{2\pi}{b}=60$，$b=\dfrac{\pi}{30}$

$y=0.5\sin\left(\dfrac{\pi}{30}t+c\right)+1.5$

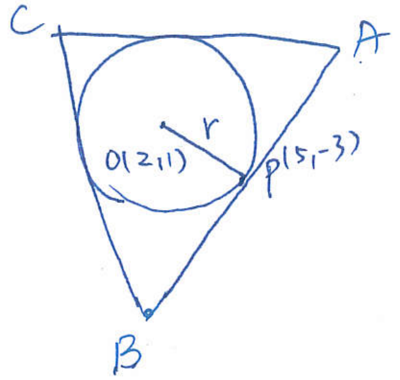
當 $t=0$ 時，有最小值，即 $\sin\left(\dfrac{\pi}{30}\cdot 0+c\right)=-1$，$c=\dfrac{-\pi}{2}$

$f(40)=0.5\sin\left(\dfrac{40}{30}\pi-\dfrac{\pi}{2}\right)+1.5=0.5\sin\left(\dfrac{5}{6}\pi\right)+1.5$

$=0.5\times\dfrac{1}{2}+1.5=\dfrac{7}{4}$

選(1)(4)(5)#

12. $\Gamma$: $(x-2)^2+(y-1)^2=5-f$




(1) $r=\sqrt{3^2+4^2}=5$

$5-f=r^2=25$, $f=-20$ (×)

(2) ∵ $\angle A=60^\circ$

∴ $\triangle AOP$ 為 $30^\circ-60^\circ-90^\circ$ 的三角形

∴ $\overline{AP}=\sqrt{3}r=5\sqrt{3}$ (○)

(3) $\overrightarrow{OP}=(3,-4)$ ∴ $\overrightarrow{PA}\parallel(4,3)$

又 $|\overrightarrow{AP}|=5\sqrt{3}$ ∴ $\overrightarrow{PA}=\pm\sqrt{3}(4,3)$

"+"為 A點 ∴ $A=P+\overrightarrow{PA}$

"−"為 B點 $=(5,-3)+(4\sqrt{3},3\sqrt{3})$

$=(5+4\sqrt{3},-3+3\sqrt{3})$ (○)

(4) 承(3)，$B=P+\overrightarrow{PB}=(5,-3)+(-4\sqrt{3},-3\sqrt{3})$

$=(5-4\sqrt{3},-3-3\sqrt{3})$ (×)

(5) ∵ O是 $\triangle ABC$ 的內心，但 $\triangle ABC$ 為正三角形

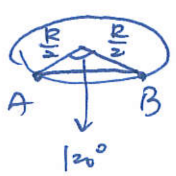
∴ O也是 $\triangle ABC$ 的重心 ∴ $\overrightarrow{OC}=2\overrightarrow{PO}=2(-3,4)=(-6,8)$

$C=O+\overrightarrow{OC}=(2,1)+(-6,8)=(-4,9)$ (○)

選 (2)(3)(5)

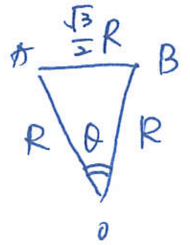
13. 設球半徑R，則北緯 $60^\circ$ 的圓半徑為 $R\cdot\cos60^\circ=\frac{1}{2}R$

考慮北緯 $60^\circ$




∴ $\overline{AB}=\frac{R}{2}\cdot\sqrt{3}$

考慮 $\triangle OAB$




$$\cos\theta=\frac{R^2+R^2-\left(\frac{\sqrt{3}}{2}R\right)^2}{2\cdot R\cdot R}=\frac{1+1-\frac{3}{4}}{2\cdot1\cdot1}=\frac{5}{8}$$ #

14、

$$f(x) = a(x-1)^3 + p(x-1) + 2$$

$$f(2) = a + p + 2 = 0 \qquad \Rightarrow \left| \begin{array}{l} a+p=-2 \\ 4a+p=1 \end{array} \right. \qquad a=1,\ p=-3$$

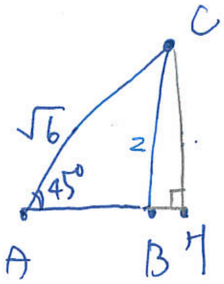
$$f(3) = 8a + 2p + 2 = 4$$

$\therefore\ g(x) = 2(x-1)^3 - 6(x-1) + 5$ 在 $x=1$ 近似直線 $y = -6(x-1)+5$

$$= -6x + 11$$

$$\underline{(p, q) = (-6, 11)}\ \#$$

15、




$\because \angle A = 45^\circ$

$\therefore \angle B > 2\angle A$, 即 $\angle B$ 為鈍角.

作 C 對 $\overleftrightarrow{AB}$ 之垂足 H

$\therefore \triangle ACH$ 為 $45^\circ-45^\circ-90^\circ$

$\therefore \overline{CH} = \overline{AH} = \sqrt{3} \qquad \therefore \overline{BH} = \sqrt{2^2 - \sqrt{3}^2} = 1$

$\therefore \overline{AB} = \sqrt{3} - 1$

$\because \angle CBH = 60^\circ, \quad \therefore \angle ABC = 120^\circ = \frac{2\pi}{3}$

$$\underline{\left(\frac{2\pi}{3}, \sqrt{3}-1\right)}\ \#$$

16、

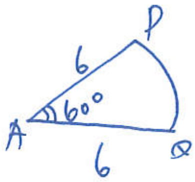
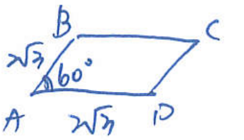
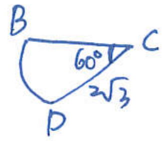
$$\overrightarrow{MG} = \overrightarrow{MA} + \overrightarrow{AG}$$

$$= -\overrightarrow{AM} + \overrightarrow{AG} = -\frac{1}{2}\overrightarrow{AD} + \overrightarrow{AG}$$

$$= -\frac{1}{2}\Big(\underbrace{\frac{2}{5}\overrightarrow{AC} + \frac{3}{5}\overrightarrow{AB}}_{\text{分點公式}}\Big) + \Big(\underbrace{\frac{1}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{AC}}_{\text{重心性質}}\Big)$$

$$= \left(\frac{1}{3} - \frac{3}{10}\right)\overrightarrow{AB} + \left(\frac{1}{3} - \frac{1}{5}\right)\overrightarrow{AC}$$

$$= \frac{1}{30}\overrightarrow{AB} + \frac{2}{15}\overrightarrow{AC} \qquad \underline{(r, s) = \left(\frac{1}{30}, \frac{2}{15}\right)}\ \#$$

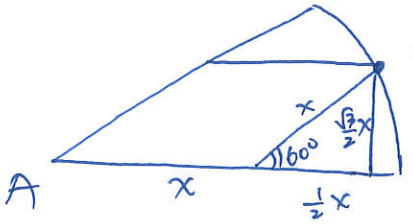
17. 所求面積 =  −  + 



$= \pi\cdot 6^2 \times \frac{60^\circ}{360^\circ} - 2\sqrt{3}\times 2\sqrt{3}\times \sin 60^\circ + \pi\cdot(2\sqrt{3})^2\cdot\frac{60^\circ}{360^\circ} = \underline{8\pi - 6\sqrt{3}}$ #

(∵ 以 C 為圓心畫 $\overparen{BD}$ ∴ $\overline{BC} = \overline{BD}$

∴ $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DA}$





$(\frac{3}{2}x, \frac{\sqrt{3}}{2}x)$

可設 $A(0,0)$、$C(\frac{3}{2}x, \frac{\sqrt{3}}{2}x)$

$\overline{AC} = r = 6 = \sqrt{\frac{9}{4}x^2 + \frac{3}{4}x^2} = \sqrt{3}x$

∴ $x = 2\sqrt{3}$ )

18. $A^2 = \begin{bmatrix}1 & a\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & a\\ 0 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I$

$B^2 = \begin{bmatrix}1 & 0\\ b & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ b & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I$

(1) $A^2 = I = B^2$ (o)　　(2) $A^3 = A \neq B = B^3$ (x)

(3) $AB = \begin{bmatrix}1 & a\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ b & -1\end{bmatrix} = \begin{bmatrix}1+ab & -a\\ -b & 1\end{bmatrix}$

$BA = \begin{bmatrix}1 & 0\\ b & -1\end{bmatrix}\begin{bmatrix}1 & a\\ 0 & -1\end{bmatrix} = \begin{bmatrix}1 & a\\ b & ab+1\end{bmatrix}$

∴ $AB \neq BA$ (x)

(4) $A^2 = I$ ∴ $A = A^{-1}$ (o)

(5) $B^{-1} = B \neq I = B^{10}$ (x)

選 $\underline{(1)(4)}$ #

19. $(A+I)^3 = A^3 + 3A^2 + 3A + I$

$= A + 3I + 3A + I = 4A + 4I$　　$\underline{(4,4)}$ #

20. ① 當 n 是奇數 $A^n = A \Rightarrow A^n X_n = B \Rightarrow AX_n = B$. $X_n = A^{-1}B$

$\Rightarrow X_n = AB = \begin{bmatrix}1+ab & -a\\ -b & 1\end{bmatrix}$

② 當 n 是偶數 $A^n = I$, $A^n X_n = B$, $IX_n = B$, $X_n = \begin{bmatrix}1 & 0\\ b & -1\end{bmatrix}$