1、

32

↓ $\quad C^4_2 \times 8 = 48$ 場

16

↓ $\quad$ 8 場（單淘汰，淘汰1隊需1場）

8

↓ $\quad$ 4 場

4

↓ $\quad$ 2 場

2

↙1 ↘1 $\quad$ 2 場 $\qquad$ 共 64 場，<u>選 (5)</u>#

舒 $\quad$ 李

2、設光度分別為 y_1, y_2，亦即 $\log y_1 = 5.3$，$\log y_2 = 2.3$

$\therefore \log y_1 - \log y_2 = 3$，$\log \frac{y_1}{y_2} = 3$，$\frac{y_1}{y_2} = 1000$，<u>選 (4)</u>#

3. $r_B = r_C = 1 > r_A = r_D = 0 > r_E = -1$，<u>選 (5)</u>#

4. $\log_3 (a_3 \cdot a_7) = 4 \Rightarrow a_3 \cdot a_7 = 3^4 = 81 \cdots ①$

$\log_2 (a_4 \cdot a_8) = 6 \Rightarrow a_4 \cdot a_8 = 2^6 = 64 \cdots ②$

$\frac{②}{①}: \left(\frac{a_4}{a_3}\right) \times \left(\frac{a_8}{a_7}\right) = \frac{64}{81}$，$r^2 = \frac{64}{81}$，$r = \frac{8}{9}$，<u>選 (5)</u>#

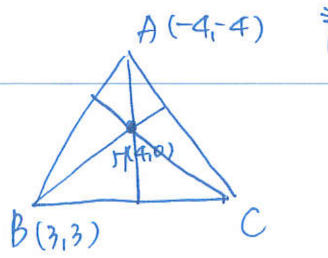
5.

	男	女
讚	186	x
沒讚	434	$380-x$
	620	380

$\frac{186}{434} = \frac{x}{380-x} \Rightarrow \frac{380-x}{x} = \frac{434}{186} = \frac{217}{93} = \frac{7}{3}$

$\Rightarrow \frac{380}{x} - 1 = \frac{7}{3}$，$\frac{380}{x} = \frac{10}{3}$，$x = 38 \times 3 = 114$，<u>選 (4)</u>#

6.



設 $C(x, y)$

$\overrightarrow{AB} \cdot \overrightarrow{CH} = 0 \Rightarrow (7,7) \cdot (4-x, -y) = 0$，$4-x-y=0$，$x+y=4$

$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0 \Rightarrow (8,4) \cdot (x-3, y-3) = 0$，$2(x-3)+y-3=0$，$2x+y=9$

$\therefore x = 5, y = -1$ $\qquad$ <u>選 (1)</u>#

7. 亦即 $A^n = I$

(1) $A^n = \begin{bmatrix} 2^n & 0 \\ 0 & (\frac{1}{2})^n \end{bmatrix} \neq I \ (n \in \mathbb{N})$

(2) A 是旋轉矩陣 $\theta = 60^\circ \Rightarrow A^6 = I$ (○)

(3) A 是旋轉矩陣 $\theta = 210^\circ \Rightarrow A^{12} = I$ (○)

(4) A 是鏡射矩陣 $A^2 = I$ (○)

(5) $A^n = \begin{bmatrix} 1 & (\frac{1}{2}) \times n \\ 0 & 1 \end{bmatrix} \neq I$

選 (2)(3)(4) #

8. 設三邊長 $x-1,\ x,\ x+1$

∵ 二邊之和 > 第三邊 ∴ $(x-1)+x > x+1$, $x > 2 \cdots ①$

又鈍角 ∴ $(x+1)^2 > (x-1)^2 + x^2$ ∴ $x^2+2x+1 > x^2-2x+1+x^2$, $x^2-4x<0$, $0<x<4 \cdots ②$

由①②知 $x=3$，亦即三邊長為 2, 3, 4

(1) 最大邊長為 4 (×)

(2) $\cos A = \dfrac{2^2+3^2-4^2}{2\cdot 2\cdot 3} = \dfrac{-3}{12} = \dfrac{-1}{4}$ (○)

(3) $\cos B = \dfrac{4^2+3^2-2^2}{2\cdot 4\cdot 3} = \dfrac{21}{24} = \dfrac{7}{8}$ (○)

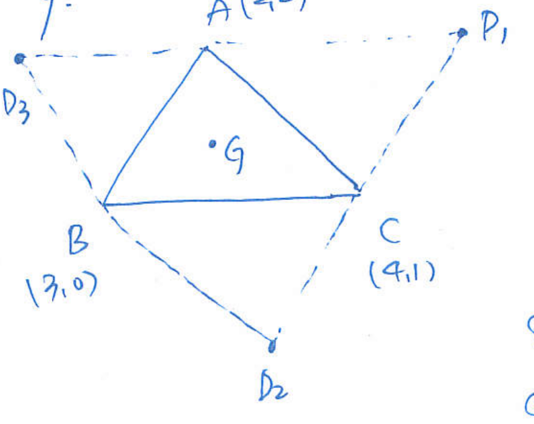
(3) $\cos A = \dfrac{-1}{4}$, $\sin A = \dfrac{\sqrt{15}}{4}$

$\triangle ABC$ 面積 $= \frac{1}{2} \times 2 \times 3 \times \dfrac{\sqrt{15}}{4} = \dfrac{3\sqrt{15}}{4}$ (○)

(5) 即求外接圓半徑: $\dfrac{4}{\frac{\sqrt{15}}{4}} = 2R$, $R = \dfrac{8}{\sqrt{15}}$ (○)

選 (2)(3)(4)(5) #

9.



可能的 D 值有四：

重心 G，及 D_1, D_2, D_3 其中 A, B, C, D 可構成平行四邊形

① $G = \dfrac{A+B+C}{3} = (3,1)$

② $D_1 = A+C-B = (3,3)$

③ $D_2 = B+C-A = (5,-1)$

④ $D_3 = A+B-C = (1,1)$

選 (2)(3)(4)(5) #

10.

(1)(2) $f(x) = 1\cdot(x-2)^3 + p(x-2) + 1$

過 $(0,-9) \Rightarrow -9 = -8 - 2p + 1$，$p = 1$

$\therefore f(x) = 1(x-2)^3 + (x-2) + 1$

$= x^3 - 6x^2 + 12x - 8 + x - 2 + 1 = x^3 - 6x^2 + 13x - 9$

(3) $f(1) = 1 - 6 + 13 - 9 = -1$ (o)

(4)
$$\begin{array}{rrrr|l} 1 & -6 & 13 & -9 & -2 \\ & -2 & 16 & -58 & \\ \hline 1 & -8 & 29 & -67 & \\ & -2 & 20 & & \\ \hline 1 & -10 & 49 & & \end{array}$$

$\therefore f(x)$ 在 $x=-2$ 近似直線 $y = 49(x+2) - 67$ (x)

(5) 三次函數的值域為所有實數 (o)

選 (1)(3)(5) #

11. 坐標化：設 $A(0,0,0)$，$C(1,0,0)$，$B(0,1,0)$

$\therefore M(1,2,-1)$，$O(1,0,-2)$

(1) $\overline{AM} = \sqrt{1^2+2^2+1^2} = \sqrt{6}$ (o)

(2)
$$\cos\angle OAM = \frac{\overrightarrow{AO}\cdot\overrightarrow{AM}}{|\overrightarrow{AO}||\overrightarrow{AM}|} = \frac{1+0+2}{\sqrt{5}\ \sqrt{6}} = \frac{3}{\sqrt{30}} \quad (o)$$

(3) $\overleftrightarrow{AJ}$、$\overleftrightarrow{LN}$ 同時平行 (xz 平面)，故沒有交點

又 $\overrightarrow{AJ} \nparallel \overrightarrow{LN}$ $\therefore \overleftrightarrow{AJ}$、$\overleftrightarrow{LN}$ 歪斜 (o)

(4) $\overleftrightarrow{AJ} \perp \overleftrightarrow{JN}$ 故 $\overline{JN}$ 為兩歪斜線 $\overleftrightarrow{AJ}$、$\overleftrightarrow{SN}$ 之公垂線段

$\overleftrightarrow{SN} \perp \overleftrightarrow{JN}$

$\therefore d(\overleftrightarrow{AJ}, \overleftrightarrow{SN}) = \overline{JN} = 3$ (o)

(5) $B(0,1,0)$，$I(0,1,-1)$，$O(1,0,-2)$

$\overrightarrow{BI} \times \overrightarrow{BO} = (0,0,-1) \times (1,-1,-2) \parallel (1,1,0)$

$$\begin{array}{cccccc} 0 & 0 & -1 & 0 & 0 & \\ 1 & -1 & -2 & 1 & -1 & \\ \hline & (-1, & -1, & 0) & & \end{array}$$

平面 BIO：$x + y = 1$

$\because$ 平面 FLQ $\parallel$ 平面 BIO：平面 FLQ：$x + y = 3$ $F(0,3,0)$

$$\therefore d(BIO, FLQ) = \frac{|3-1|}{\sqrt{1^2+1^2}} = \sqrt{2} \quad (x)$$

選 (5) #

12. (1) $f(x)$ 除以 $(x-1)(x-2)$ 餘 $ax+b$: $\begin{cases} f(1)=a+b=5 \\ f(2)=2a+b=8 \end{cases}$, $a=3, b=2$ 餘 $3x+2$ (x)

(2) $f(x)$ 除以 $(x-1)(x-3)$ 餘 $cx+d$: $\begin{cases} f(1)=c+d=5 \\ f(3)=3c+d=13 \end{cases}$, $c=4, d=1$ 餘 $4x+1$ (o)

(3) $f(x)$ 除以 $(x-2)(x-3)$ 餘 $ex+f$: $\begin{cases} f(2)=2e+f=8 \\ f(3)=3e+f=13 \end{cases}$, $e=5, f=-2$ 餘 $5x-2$ (o)

(4) $f(x)=(x-2)(x-3)\cdot Q(x)+5x-2$
$=[2(x-2)(x-3)]\cdot\frac{Q(x)}{2}+5x-2$ ∴餘 $5x-2$ (o)

(5) $f(1)=5\neq f$ (x)

選 (2)(3)(4) #

13. $\pi\cdot 400^2\times\frac{120^\circ}{360^\circ}-90\times 90\times\sin 120^\circ=\frac{160000}{3}\pi-4050\sqrt{3}$ #

14.
$600\times\frac{1}{2}+500\times\frac{1}{2}\times\frac{1}{2}+400\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+300\times(\frac{1}{2})^4+200\times(\frac{1}{2})^5+100\times(\frac{1}{2})^6+0\times(\frac{1}{2})^6$

$=300+125+50+18.75+6.25+1.5625=501.5625$

15. $\begin{cases} (1-k)x+2y-z=0 \cdots ① \\ (k-2)x+y+3z=0 \cdots ② \\ 2x+4y+kz=0 \cdots ③ \end{cases}$ 有無限多解

②×2−①: $(3k-5)x+7z=0$
①×2−③: $(-2k)x+(-2-k)z=0$

∴ $\frac{3k-5}{-2k}=\frac{7}{-2-k}$
$\Rightarrow -3k^2-k+10=-14k$

$3k^2-13k-10=0 \Rightarrow (3k+2)(k-5)=0$, $k=\frac{-2}{3}$ or 5

∵ $k\in\mathbb{Z}$ ∴ $k=5$ #

16. 令 $\sin\theta+\cos\theta=t$, $(\sin\theta+\cos\theta)^2=t^2 \Rightarrow t^2=1+2\sin\theta\cdot\cos\theta$

$\therefore \sin\theta\cdot\cos\theta=\dfrac{t^2-1}{2}$

$$f(\theta)=\frac{\frac{t^2-1}{2}}{1+t}=\frac{t-1}{2}\le\frac{\sqrt{2}-1}{2}$$

($\because \sin\theta+\cos\theta=\sqrt{2}\sin(\theta+45^\circ)$, 其中 $45^\circ\le\theta+45^\circ\le 225^\circ$

$\therefore -1\le t=\sin\theta+\cos\theta\le\sqrt{2}$)

17. $\begin{vmatrix} a & -b & c \\ 2 & 1 & 2 \\ -p & q & -r \end{vmatrix}$ 可想成 $(a,-b,c)$, $(2,1,2)$, $(-p,q,-r)$ 所展出平行六面體體積

故最大值為 $\sqrt{169}\times\sqrt{2^2+1^2+2^2}\times\sqrt{100}=13\times3\times10=\underline{390}$

18. 最高點會越來越大, 但最低點不變, $\underline{\text{選(5)}}$

19. 上午+星光 最多玩 5 個, 故下午必為剩餘 3 個 (A, D, H)

註: 將設施由上到下編 A~H

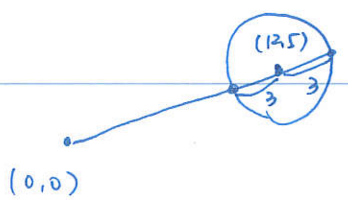
E 必為上午, B 必為星光, 剩下 C, F, G 可 2 個上午, 1 個星光 …①

或 1 個上午, 2 個星光 …②

	上午	下午	星光	
case 1:	$C^3_2\times3!$	$\times3!$	$\times2!$	$=216$
case 2:	$C^3_1\times2!$	$\times3!$	$\times3!$	$=216$

共 432 種

20. $(x-12)^2+(y-5)^2=-160+144+25=9$



$\sqrt{12^2+5^2}=13$

d: $13-3 \sim 13+3$

$\therefore |d-10|\le3$

$\underline{(a,b)=(13,3)}$