

1. 期望值即平均。

$$\frac{3 \times 50 + 2 \times 100}{5} = 70$$

(1) *

2.

$$\begin{aligned} & 4(x^2+1) + (x+1)^2(x-3) + (x-1)^3 \\ &= 4x^2+4 + (x^2+2x+1)(x-3) + (x^3-3x^2+3x-1) \\ &= \underline{4x^2+4} + \underline{x^3+2x^2+x} - \underline{3x^2-6x-3} + \underline{x^3-3x^2+3x-1} \\ &= 2x^3-2x = 2x(x^2-1) = 2x(x-1)(x+1) \end{aligned}$$

(5) *

$$3. (a_{n+1})^2 = \frac{1}{\sqrt{10}} (a_n)^2 \Rightarrow \left(\frac{a_{n+1}}{a_n}\right)^2 = \frac{1}{\sqrt{10}}$$

$$\text{取 } \log \Rightarrow \log \left(\frac{a_{n+1}}{a_n}\right)^2 = \log \frac{1}{\sqrt{10}} \Rightarrow 2(\log a_{n+1} - \log a_n) = \frac{-1}{2}$$

$$\therefore \log a_{n+1} - \log a_n = \frac{-1}{4} \Rightarrow b_{n+1} - b_n = \frac{-1}{4}$$

後前 - 前 = 負一定值，即為等差數列，公差 $\frac{-1}{4}$ (2) *

$$4. \left(\frac{x^2}{5^2} + \frac{y^2}{4^2}\right) \left(\frac{x^2}{3^2} - \frac{y^2}{4^2}\right) = 0$$

$$\Rightarrow \left(\frac{x^2}{5^2} + \frac{y^2}{4^2}\right) \left(\frac{x}{3} - \frac{y}{4}\right) \left(\frac{x}{3} + \frac{y}{4}\right) = 0$$

$$\therefore \frac{x^2}{5^2} + \frac{y^2}{4^2} = 0 \quad \text{or} \quad \frac{x}{3} - \frac{y}{4} = 0 \quad \text{or} \quad \frac{x}{3} + \frac{y}{4} = 0$$

是 (0,0)

直線

直線

→ (0,0) ←

(3) *

5.

$$(1) \quad 3^7 = 2187 \quad \therefore 3^7 > 7^3 \quad (x)$$

$$7^3 = 243$$

$$(2) \quad 5^{10}, 10^5$$

* 次方無法處理 \Rightarrow 取 \log .

$$\log 5^{10} = 10 \log 5 = 10 \times 0.699 = 6.99 \quad \therefore 5^{10} > 10^5 \quad (x)$$

$$\log 10^5 = 5$$

(3) 同上. 次方無法處理 \Rightarrow 取 \log .

$$\log 2^{100} = 100 \cdot \log 2 = 30.1 \quad \therefore 2^{100} > 10^{30} \quad (x)$$

$$\log 10^{30} = 30$$

$$(4) \quad \log_2 3 = \frac{\log 3}{\log 2} = \frac{0.4771}{0.3010} \neq 1.5 \quad (x)$$

$$(5) \quad \log_2 11 < 3.5 \quad \text{可想成} \quad 11 < 2^{3.5} \quad (\text{同對或同錯})$$

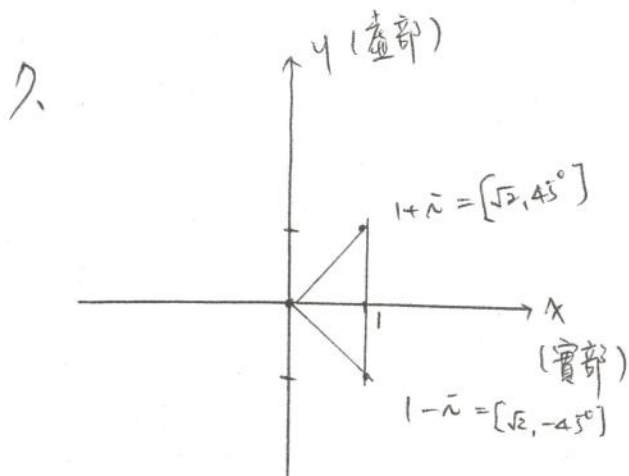
$$2^{3.5} = 8\sqrt{2} = 8 \times 1.414 = 11.312 \quad (0)$$

(5) *

6. 61個數的中位數為第 $\frac{61+1}{2} = 31$ 個數.

第25個數	0.2019
26	0.2034
27	0.2051
28	0.2085
29	0.2123
30	0.2137
31	0.2164

(2) #



$$(1) \cos 60^\circ = \left[\frac{1}{2}, 0^\circ \right] \quad (0)$$

$$(2) \cos 50^\circ + i \sin 50^\circ = [1, 50^\circ] \quad (x)$$

$$(3) \frac{4-3i}{5} = \frac{4}{5} + \frac{-3}{5}i = [1, -37^\circ] \quad (0)$$



$$(4) \frac{1+\sqrt{3}i}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2}i = [1, 60^\circ] \quad (x)$$

$$(5) (\cos 30^\circ + i \sin 30^\circ)^5 = \cos 750^\circ + i \sin 750^\circ = [1, 30^\circ] \quad (0)$$

$$= \cos 30^\circ + i \sin 30^\circ$$

(1)(3)(5) #

8.

$$\sin \theta = -\frac{2}{3}, \cos \theta > 0 \Rightarrow \theta \in \text{IV}, \quad \frac{3}{\sqrt{5}}$$

$$(1) \tan \theta < 0 \quad (0)$$

$$(2) \tan^2 \theta = \left(-\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5} > \frac{4}{9} \quad (0)$$

$$(3) \sin^2 \theta = \frac{4}{9}$$

$$> \sin^2 \theta < \cos^2 \theta \quad (x)$$

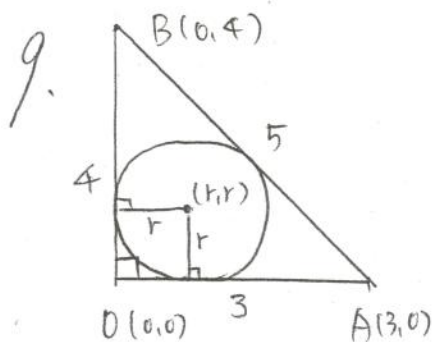
$$\cos^2 \theta = \frac{5}{9}$$

$$(4) \sin 2\theta = 2 \sin \theta \cdot \cos \theta = 2 \left(-\frac{2}{3}\right) \left(\frac{\sqrt{5}}{3}\right) = \frac{-4\sqrt{5}}{9} < 0 \quad (x)$$

$$(5) \because |\tan \theta| < 1 \quad \therefore \underline{-45^\circ + 360^\circ k} < \theta < \underline{0^\circ + 360^\circ k} \Rightarrow \underline{-90^\circ + 720^\circ k} < 2\theta < \underline{0^\circ + 720^\circ k}$$

$\therefore 0, 2\theta$ 均在第四象限 (x)

(1)(2) #



(1) 外接圓半徑 (R)

* 與 $\triangle ABC$ 面積有關之公式

$$\triangle ABC = \frac{1}{2} \times \text{底} \times \text{高} = \frac{1}{2} ab \sin C = \frac{abc}{4R} = r \cdot S = \sqrt{s(s-a)(s-b)(s-c)}$$

(其中 $s = \frac{a+b+c}{2}$)

$$\therefore \triangle OAB \text{ 面積} = \frac{1}{2} \times 3 \times 4 = \frac{3 \times 4 \times 5}{4R} \Rightarrow R = \frac{5}{2} \quad (x)$$

(2) 外接圓圓心為中垂線之交點

$$\overline{OA} \text{ 之中垂線: } x = \frac{3}{2} \Rightarrow \text{外心} \left(\frac{3}{2}, 2 \right)$$

$$\overline{OB} \text{ 之中垂線: } y = 2$$

\therefore 不在 $y=x$ 上 (x)

(3) $4\left(\frac{3}{2}\right) + 3 \cdot 2 = 12 \therefore$ 外心在 $4x+3y=12$ 上 (0)

(4) 內切圓半徑 (r)

$$\Rightarrow \text{內切圓圓心} (r, r)$$

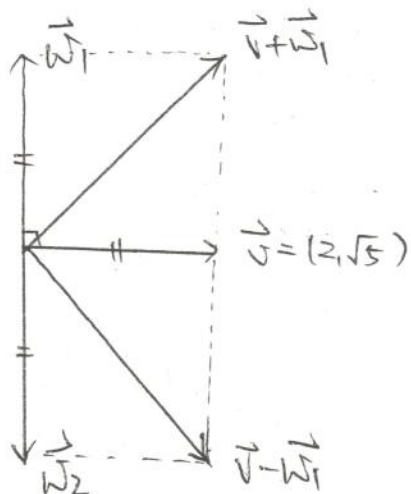
$$\triangle OAB \text{ 面積} = \frac{1}{2} \cdot 3 \cdot 4 = r \cdot \left(\frac{3+4+5}{2} \right) \Rightarrow r = 1$$

\Rightarrow 內心 (1,1), 在 $y=x$ 上 (0)

(5) $4 \cdot 1 + 3 \cdot 1 = 7 \neq 6 \therefore$ 不在 $4x+3y=6$ 上 (x)

(3)(4) ✖

10.

(1) \vec{w} 有兩種可能

$$\Rightarrow \vec{w} = \pm(\sqrt{5}, -2)$$

$$= (\sqrt{5}, -2) \text{ or } (-\sqrt{5}, 2) \quad (0)$$

(2) 如左圖，由向量加法可知。

$$|\vec{v} + \vec{w}| = |\vec{v} - \vec{w}| = \sqrt{2}|\vec{v}| \quad (0)$$

(3) $\vec{v} + \vec{w}$ 與 \vec{w} 夾角為 45° (x)

$$(4) \vec{u} = a\vec{v} + b\vec{w} \Rightarrow |\vec{u}| = |a\vec{v} + b\vec{w}|$$

(* 看到求 $|\vec{u}|$ ，想到 $|\vec{u}|^2$)

$$\begin{aligned} \therefore |\vec{u}|^2 &= |a\vec{v} + b\vec{w}|^2 = a^2|\vec{v}|^2 + 2ab\vec{v} \cdot \vec{w} + b^2|\vec{w}|^2 \\ &= a^2|\vec{v}|^2 + b^2|\vec{w}|^2 \quad (\because \vec{v}, \vec{w} \text{ 垂直} \therefore \vec{v} \cdot \vec{w} = 0) \\ &= 9(a^2 + b^2) \end{aligned}$$

$$\therefore |\vec{u}| = 3\sqrt{a^2 + b^2} \quad (x)$$

(5) ① case 1: $\vec{w} = (\sqrt{5}, -2)$

$$(1, 0) = c(2, \sqrt{5}) + d(\sqrt{5}, -2) \quad \begin{cases} 2c + \sqrt{5}d = 1 \\ \sqrt{5}c - 2d = 0 \end{cases} \Rightarrow c, d \text{ 同符} \Rightarrow c, d \text{ 同}$$

② case 2: $\vec{w} = (-\sqrt{5}, 2)$

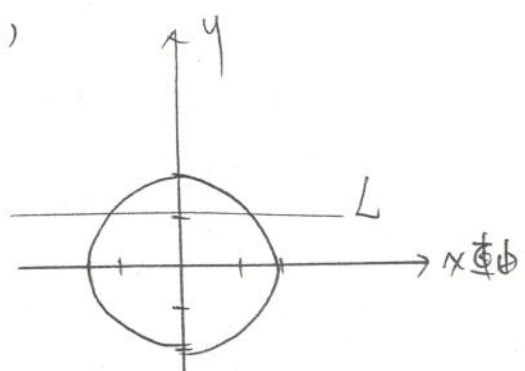
$$(1, 0) = c(2, \sqrt{5}) + d(-\sqrt{5}, 2) \quad \begin{cases} 2c - \sqrt{5}d = 1 \\ \sqrt{5}c + 2d = 0 \end{cases} \Rightarrow 2c + \frac{5}{2}c = 1 \Rightarrow c > 0$$

$$\Rightarrow d = -\frac{\sqrt{5}}{2}c \quad (0)$$

(1)(2)(5) *

11.

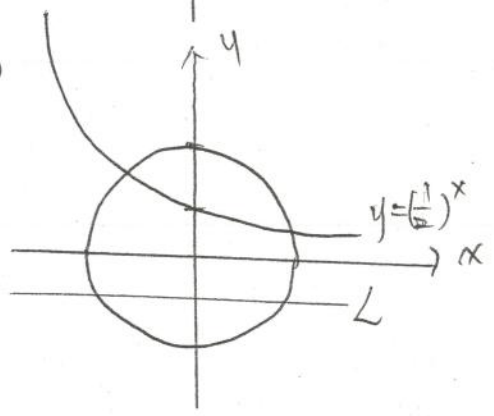
(1)



取 L 平行 x 軸 (如: $y=1$)

即可不相交於 x 軸. (X)

(2)

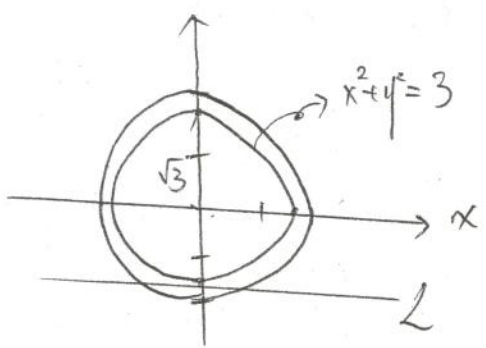


取 L 平行 x 軸 且在 x 軸 下

(如: $y=-1$)

即可不相交於 $y = (\frac{1}{2})^x$ (X)

(3)

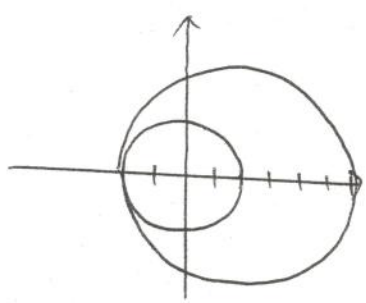


取 L 平行 x 軸 且在小圓與大圓之間

(如: $y=-1.8$)

即可不相交於 $x^2 + y^2 = 3$ (X)

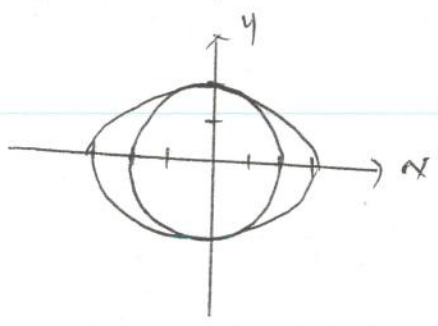
(4)



$$(x-2)^2 + y^2 = 16 \text{ 及 } \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\text{均包含 } x^2 + y^2 = 4$$

∴ 直線 L 必與 (4)(5) 圖 相交



(4)(5) #

12.

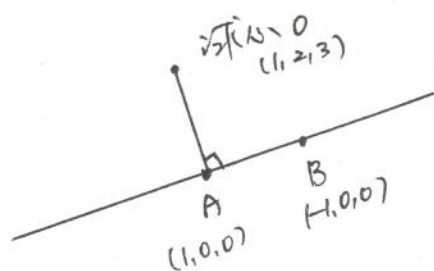
(1) $(0,0,0)$ 代入 $\Rightarrow (-1)^2 + (-2)^2 + (-3)^2 = 14 \quad \therefore (0,0,0)$ 在球面 S 上 (0)

(2) $(1,0,0)$ 代入 $\Rightarrow 0^2 + (-2)^2 + (-3)^2 = 13 < 14 \quad \therefore (1,0,0)$ 在球面 S 內部 (x)

(3) $B(-1,0,0)$ 代入 $\Rightarrow (-2)^2 + (-2)^2 + (-3)^2 = 17 > 14 \quad \therefore B$ 在球面 S 外部

$\therefore A, B$ 在球內、外 $\therefore \overline{AB}$ 必與球面 S 相交 (0)

(4) 求直線之最短距離 \Rightarrow 必定垂直



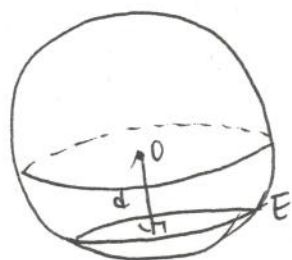
\therefore 若 A 為直線 AB 上距離至球心最近

$\Rightarrow \vec{AO} \perp \vec{AB}$

$\vec{AO} = (0, 2, 3) \quad \therefore \vec{AO} \cdot \vec{AB} = 0 \quad (0)$

$\vec{AB} = (-2, 0, 0)$

(5)



球心至平面距離越小，則截面面積越大。

(d)

球心到 xy 平面之距 $d_{xy} = 3$

$y \perp z \quad d_{yz} = 1$

$z \perp x \quad d_{zx} = 2$

\therefore 與 yz 所截之圓面積最大 (x)

“(3)(4) *

13.

$f(x) = x(x-1)(x+1)$

(1) $f(\frac{1}{\sqrt{2}}) = \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}-1)(\frac{1}{\sqrt{2}}+1) < 0 \quad (x)$

(2) $f(x) = 2 \Rightarrow x(x-1)(x+1) = 2 \Rightarrow x^3 - x = 2 \Rightarrow x^3 - x - 2 = 0$

若有整數解必為 ± 1 或 ± 2 (整係數一次因式檢驗法)

$x = \pm 1, \pm 2$ 代入均不滿足方程式 $\Rightarrow f(x) = 2$ 無整數解 (x)

$$(3) f(x) = x^2 + 1 \Rightarrow \underbrace{f(x) - x^2 - 1 = 0}_{\text{三次式}} \Rightarrow \text{必有實根 (虛根共軛)} \quad (0)$$

$$(4) f(x) = x \Rightarrow x(x-1)(x+1) = x \Rightarrow x(x-1)(x+1) - x = 0 \\ \Rightarrow x[(x-1)(x+1) - 1] = 0 \Rightarrow x(x^2 - 2) = 0 \\ \Rightarrow x = 0, \pm\sqrt{2} \quad (x)$$

$$(5) f(a) = a(a-1)(a+1) = 2 \\ f(-a) = -a(-a-1)(-a+1) \\ = -[a(a+1)(a-1)] = -f(a) = -2 \quad (X) \quad (3) \quad \#$$

第貳部分：選填題

A. 首項 a , 公比 r ($-1 < r < 1$) 之無窮等比級數和 $= \frac{a}{1-r}$

$$\begin{cases} \frac{a}{1-r} = 5 & \text{--- ①} \\ \frac{a}{1-3r} = 7 & \text{--- ②} \end{cases}$$

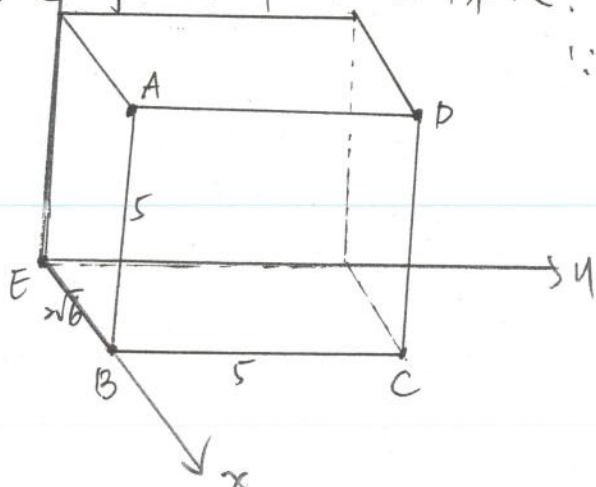
$$\frac{\text{①}}{\text{②}} \Rightarrow \frac{1-3r}{1-r} = \frac{5}{7} \Rightarrow 7-2|r = 5-5r$$

$$\Rightarrow 16r = 2 \Rightarrow r = \frac{1}{8}, a = \frac{35}{8}$$

$$\therefore \frac{a}{1-2r} = \frac{\frac{35}{8}}{1-\frac{2}{8}} = \frac{35}{6}$$

$$\frac{35}{6} \quad \#$$

B. 看到立方體 \Rightarrow 坐標化.



$$\because \cot \angle AEB = \frac{2\sqrt{6}}{5}, \text{ 設 } \overline{AB} = 5, \overline{BE} = 2\sqrt{6}$$

$$E(0,0,0), C(2\sqrt{6}, 5, 0), D(2\sqrt{6}, 5, 5)$$

$$\cos \angle CED = \frac{\vec{EC} \cdot \vec{ED}}{|\vec{EC}| |\vec{ED}|} = \frac{24 + 25 + 0}{\sqrt{49} \sqrt{74}} = \frac{7}{\sqrt{74}}$$

$$\cot \angle CED = \frac{7}{5}$$

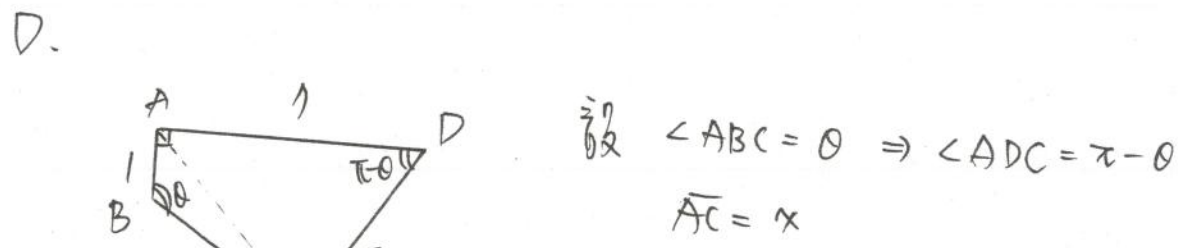
$$\frac{7}{5} \quad \#$$

C.

$$P(A) = \frac{n(A)}{n(S)} = \frac{C_2^{20} C_1^{15} + C_1^{20} C_2^{15}}{C_3^{35}} = \frac{\frac{20 \times 19}{2} \times 15 + 20 \times \frac{15 \times 14}{2}}{\frac{35 \times 34 \times 33}{1 \times 2 \times 3}} = \frac{20 \times 19 \times 15 + 20 \times 15 \times 14}{35 \times 34 \times 11}$$

$$= \frac{\overset{10}{20} \overset{3}{15} \overset{3}{(19+14)}}{\cancel{35} \times \cancel{34} \times \cancel{11}} = \frac{90}{119}$$

90
119 #



$$\cos \theta = \frac{1^2 + 5^2 - x^2}{2 \cdot 1 \cdot 5} \quad (\triangle ABC)$$

$$\cos(\pi - \theta) = \frac{7^2 + 5^2 - x^2}{2 \cdot 7 \cdot 5} \quad (\triangle ADC)$$

$$\therefore \cos(\pi - \theta) = -\cos \theta \Rightarrow \frac{7^2 + 5^2 - x^2}{2 \cdot 7 \cdot 5} = -\frac{1^2 + 5^2 - x^2}{2 \cdot 1 \cdot 5}$$

$$\Rightarrow (49 + 25 - x^2) = -(1 + 25 - x^2)$$

$$\Rightarrow 74 - x^2 = -18 + x^2 \Rightarrow 92 = 2x^2 \Rightarrow x^2 = 46$$

$$\Rightarrow x^2 = 32 \Rightarrow x = \sqrt{32}$$

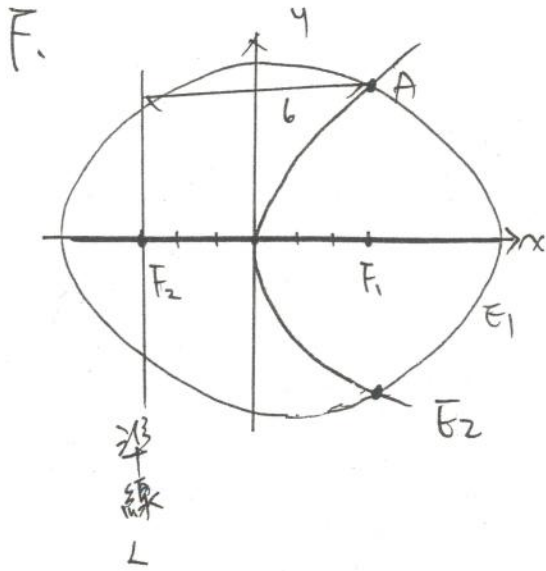
$\sqrt{32}$ #

E. 設 A, B, C 目前存 x, y, z 公克
 半年前有 2x, 3y, 4z.
 一年前有 4x, 9y, 16z

$$\begin{cases} x + 2y + z = 8 & \text{--- ①} \\ 2x + 6y + 4z = 22 & \text{--- ②} \\ 4x + 18y + 16z = 66 & \text{--- ③} \end{cases}$$

② - ① × 2: $2y + 2z = 6 \Rightarrow y = 1, z = 2, x = 4$
 ③ - ① × 4: $10y + 12z = 34$

4, 1, 2 #



設 A 為其中一點

$$\because A \text{ 在拋物線上}, \therefore \overline{AF} = d(A, L) = 6$$

$$\text{又兩焦點 } F_1 F_2 = 6$$

$$\therefore \overline{AF_2} = \sqrt{6^2 + 6^2} = 6\sqrt{2}$$

$$2a = \overline{AF_1} + \overline{AF_2} = 6 + 6\sqrt{2}$$

$$\therefore a = 3 + 3\sqrt{2}$$

$$\underline{3 + 3\sqrt{2}} \quad \#$$

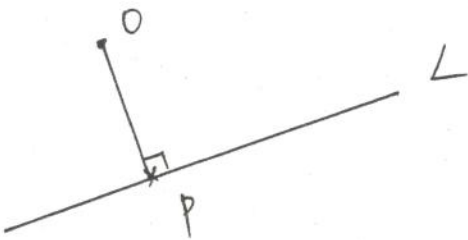
9. 設 $\vec{L} = (2, x, y)$

$$\because L \text{ 在 } E \text{ 上} \therefore \vec{L} \perp \vec{n} \Rightarrow (2, x, y) \cdot (1, -1, 1) = 0 \Rightarrow 2 - x + y = 0$$

$$\text{又 } P(2, 1, 1) \text{ 為 } L \text{ 上之一點} \therefore L \text{ 之參數式} = (2 + 2t, 1 + xt, 1 + yt), t \in \mathbb{R}$$

$$P \text{ 為原點 } O \text{ 上最近點} \Rightarrow \overline{PO} \perp \vec{L} \Rightarrow (2, 1, 1) \cdot (2, x, y) = 0$$

$$\Rightarrow 4 + x + y = 0$$



$$\begin{cases} x - y = 2 \\ x + y = -4 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = -3 \end{cases}$$

$$\underline{(2, -1, -3)} \quad \#$$