

- 1.
- (1) 小文可能英文 70 分以上, 只是數學不及格.
 - (2) 小文可能數學有及格, 只是英文未達 70 分.
 - (3) 小文可能數學有及格, 只是英文未達 70 分.
 - (4) 只要有一項 (英文未達 70 分, 數學不及格) 就不符合參選模範生
 - (5) 正確

(5) #

2.

$$a = 2.6^{10} - 2.6^9 > 0$$

$$b = 2.6^{11} - 2.6^{10} = 2.6(2.6^{10} - 2.6^9) = 2.6a$$

$$c = \frac{2.6^{11} - 2.6^9}{2} = \frac{2.6^{11} - 2.6^{10} + (2.6^{10} - 2.6^9)}{2} = \frac{a+b}{2} = \frac{3.6a}{2} = 1.8a$$

$$\therefore \underbrace{a}_{c} < 1.8a < \underbrace{2.6a}_b$$

(4) #

3. 機率: 所有物品均視為 "相異" 物.

$$P = \frac{P(\text{甲乙同色, 且兩是白色})}{P(\text{甲乙同色})}$$

$$P(\text{甲乙同色}) = \frac{n(\text{甲乙同})}{n(S)} = \frac{\overset{\text{白}}{3} \times \overset{\text{白}}{2} + \overset{\text{黑}}{2} \times \overset{\text{黑}}{1}}{5 \times 4} = \frac{8}{20}$$

$$P(\text{甲乙同色, 且兩是白色}) = \frac{n(\text{甲乙同色, 兩是白色})}{n(S)} = \frac{\overset{\text{白}}{3} \times \overset{\text{白}}{2} + \overset{\text{黑}}{2} \times \overset{\text{白}}{1}}{5 \times 4 \times 3} = \frac{12}{60}$$

$$\therefore P = \frac{\frac{12}{60}}{\frac{8}{20}} = \frac{1}{2}$$

(3) #

4. 相關係數最小, 且皆為負相關 \Rightarrow 即找相關程度最 "大" (負最多)
不難發現, x 均為 (2, 3, 5) $\therefore y$ 應由大至小才會負最多, 如 $\uparrow \dots \rightarrow$
僅 (5) 符合此規則

(5) #

5. 黃、綠為正奇數 \Rightarrow 紅必為正偶數.

設黃袋有 $2x+1$ 個球
綠袋有 $2y+1$ 個球
紅袋有 $2z+2$ 個球

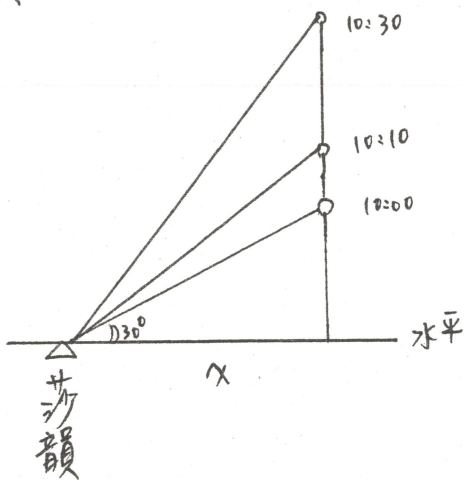
其中 x, y, z 均為非負整數.

$$\Rightarrow (2x+1) + (2y+1) + (2z+2) = 24 \Rightarrow 2(x+y+z) = 20$$

$$\Rightarrow x+y+z=10 \Rightarrow H_{10}^3 = C_{10}^{12} = C_2^{12} = \frac{12 \times 11}{2 \times 1} = 66$$

(2) #

6.



設莎韻望熱氣球水平距為 x

$$\Rightarrow 10:00 \text{ 熱氣球高度為 } x \cdot \tan 30^\circ = 0.577x$$

$$\Rightarrow 10:10 \text{ 熱氣球高度為 } x \cdot \tan 34^\circ = 0.675x$$

$$\therefore 10 \text{ 分鐘上升 } 0.098x$$

$$\Rightarrow 10:30 \text{ 熱氣球高度為 } 0.675x + 0.098x \cdot 2 = 0.871x \approx x \cdot \tan 41^\circ$$

(3) #

7. $n=2 \Rightarrow$ 找規律

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^3 = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 0 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^4 = \begin{bmatrix} 1 & 7 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 0 & 16 \end{bmatrix}$$

$$\Rightarrow \langle a_n \rangle = 1, 1, 1, \dots = \langle 1 \rangle$$

$$\langle b_n \rangle = 1, 3, 7, 15, \dots = \langle 2^n - 1 \rangle$$

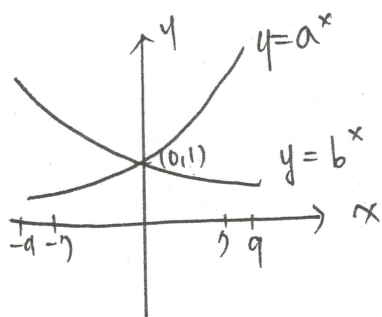
$$\langle c_n \rangle = 0, 0, 0, \dots = \langle 0 \rangle$$

$$\langle d_n \rangle = 2, 4, 8, 16, \dots = \langle 2^n \rangle$$

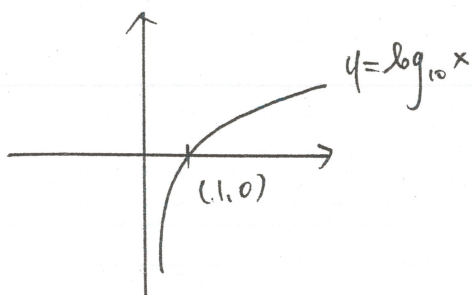
(1)(2)(3)(5) #

8. (1) $a > 1 \Rightarrow a^7 < a^9 \Rightarrow -a^7 > -a^9$ (0)

(2) $0 < b < 1 \Rightarrow b^{-7} > b^{-9}$ (0)



(3) $\because a > 1 > b \Rightarrow \frac{1}{a} < 1 < \frac{1}{b} \Rightarrow \log_{10} \frac{1}{a} < \log_{10} \frac{1}{b}$ (x)



(4) $\log_a 1 = \log_b 1 = 0$ (x)

(5) $\log_a b = \frac{\log_{10} b}{\log_{10} a} < 1$ $\therefore \log_a b < \log_b a$ (x)

$\log_b a = \frac{\log_{10} a}{\log_{10} b} > 1$

$(\because a > b \Rightarrow \log_{10} a > \log_{10} b)$

(1)(2) #

9. $f(x)$ 通過 $(a, 0), (b, 0) \Rightarrow f(x) = k_1(x-a)(x-b)$ ($k_1 > 0$)

$g(x)$ 通過 $(b, 0), (c, 0) \Rightarrow g(x) = k_2(x-b)(x-c)$ ($k_2 > 0$)

$\Rightarrow f(x) + g(x) = k_1(x-a)(x-b) + k_2(x-b)(x-c)$
 $= (x-b)[(k_1+k_2)x - (ak_1+ck_2)]$

$\therefore f(x) + g(x) = 0 \Rightarrow x = b, \frac{ak_1+ck_2}{k_1+k_2}$

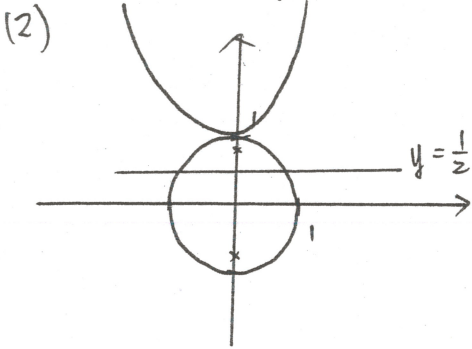
必介於 a, c 之間 (可能是 b)

(4)(5) #

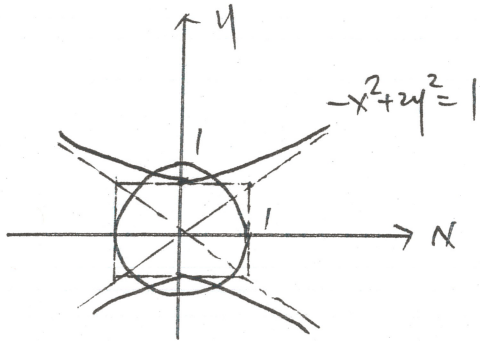
10. 設動點 $P(x, y)$

$\Rightarrow \vec{PA}_1 \cdot \vec{PA}_2 < 0 \Rightarrow (x-1, y) \cdot (x+1, y) < 0$

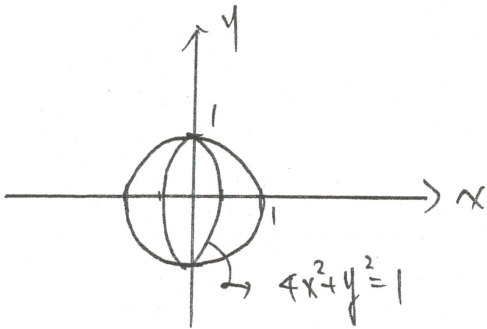
(1) $\Rightarrow x^2 - 1 + y^2 < 0 \Rightarrow x^2 + y^2 < 1$ (單位圓內部)



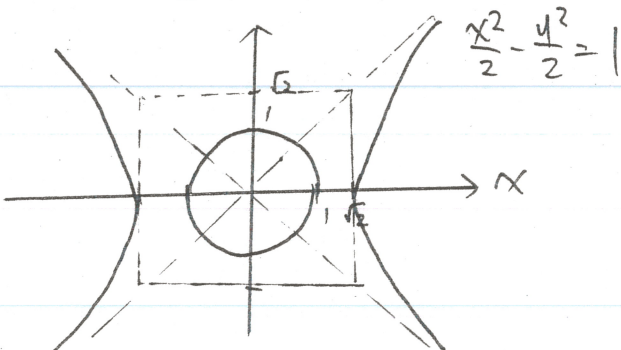
(3) $-x^2 + 2y^2 = 1 \Rightarrow -\frac{x^2}{1} + \frac{y^2}{\frac{1}{2}} = 1$ 上下型, $a = \frac{1}{\sqrt{2}}, b = 1$, (0)
中心 (0,0) 頂點 $(0, \pm \frac{1}{\sqrt{2}})$



(4) $4x^2 + y^2 = 1 \Rightarrow \frac{x^2}{\frac{1}{4}} + \frac{y^2}{1} = 1 \Rightarrow$ 上下型 $a = 1, b = \frac{1}{2}$ (0)
中心 (0,0).



(5) $\frac{x^2}{2} - \frac{y^2}{2} = 1 \Rightarrow$ 左右型 $a = \sqrt{2}, b = \sqrt{2}$, 中心 (0,0) (x)



(1)(3)(4) #

11.

① 若 S 的各邊平行軸 $\Rightarrow 0, 2$ ② 若 S 的各邊不平行軸 $\Rightarrow 0, 1, 2$ (1)(2)(5) *

12.

① $\because r = -0.8 \therefore a_9, a_{10}$ 必一正一負 $\therefore a_9 \cdot a_{10} < 0$ (0)② $\because a_9 > b_9, a_{10} > b_{10}$, 又 a_9, a_{10} 其一為負. $\therefore b_9, b_{10}$ 至少有一負.case 1: $b_9 < 0$, 又 $b_1 = 10 \therefore d < 0 \Rightarrow b_{10} < 0$. \rightarrow (x)case 2: $b_{10} < 0$ ③) 由已知 $d < 0 \Rightarrow b_9 > b_{10}$ (0)④) a_9, a_{10} 無絕對大小關係.⑤) 若 $a_9 > 0 \Rightarrow a_8 < 0$, 但 b_8 可能大於 0 $\therefore a_8, b_8$ 無絕對大小關係(1)(3) *

A. $\frac{k}{3} < \sqrt{31} < \frac{k+1}{3} \Rightarrow k < 3\sqrt{31} = \sqrt{279} < k+1$

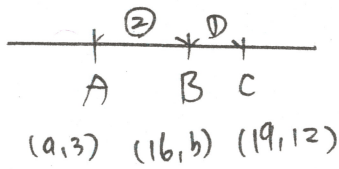
$\sqrt{279} = 16. \dots \Rightarrow k = \underline{16}$ *

B. $(a+bi)(2+6i) = (2a-6b) + (6a+2b)i = -80 + 0i$

$$\begin{cases} 2a-6b = -80 & \Rightarrow a-3b = -40 & \Rightarrow \underline{a = -4, b = 12} \end{cases} *$$

$$\begin{cases} 6a+2b = 0 & \Rightarrow b = -3a \end{cases}$$

C.



$$\therefore \vec{AC} = 3\vec{BC}$$

$$\Rightarrow (19-a, 9) = 3(3, 12-b)$$

$$\begin{cases} 19-a=9 & \Rightarrow a=10 \\ 9=36-3b & \Rightarrow b=9 \end{cases}$$

$$a+b = \underline{19}^{\#}$$

D. 第三天賣 t 公斤第二天賣 $at+b$ 公斤

第一天賣 $100-t-(at+b)$ 公斤
 $= 100-b-(a+1)t$

註: t 是變數,
 a, b 為定值.

$$\therefore [100-b-(a+1)t] \cdot 40 + (at+b) \cdot 36 + 32t = 3720$$

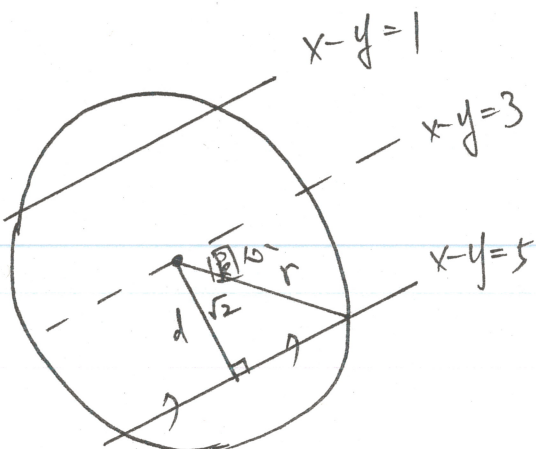
$$\Rightarrow \underline{4000} - \underline{40b} - \underline{40(a+1)t} + \underline{36at} + \underline{36b} + \underline{32t} = \underline{3720}$$

$$\Rightarrow 280 - 4at - 4b - 8t = 0 \quad (\text{對 } t \text{ 整理})$$

$$\Rightarrow (4a+8)t = 280 - 4b$$

$$\therefore \begin{cases} 4a+8=0 \\ 280-4b=0 \end{cases} \Rightarrow \underline{a=-2, b=70}^{\#}$$

E.



$\therefore x-y=1, x-y=5$ 為兩平行線

\therefore 圓心在 $x-y=3$ 上

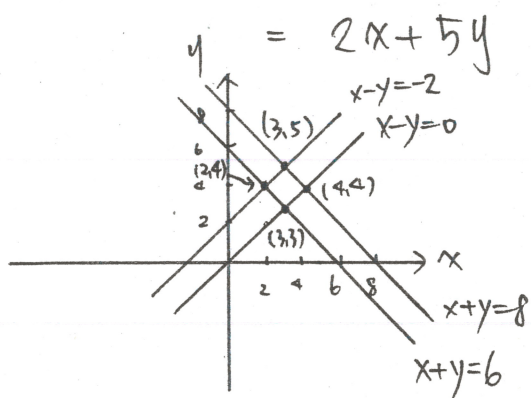
$$d = \frac{2}{\sqrt{1^2+(-1)^2}} = \sqrt{2}$$

$$r = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$$

$$\underline{\text{圓面積}} = \underline{3\pi}^{\#}$$

F.

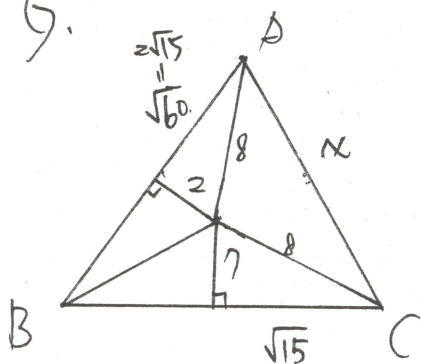
$$\begin{aligned}\vec{u} \cdot \vec{v} &= (\vec{A} + \vec{B}) \cdot (x\vec{A} + y\vec{B}) \\ &= x|\vec{A}|^2 + (x+y)\vec{A} \cdot \vec{B} + y|\vec{B}|^2 \\ &= x \cdot 1^2 + (x+y) \cdot 1 \cdot 2 \cdot \cos_{\frac{1}{2}} 60^\circ + y \cdot 2^2\end{aligned}$$



頂點	(2,4)	(3,3)	(4,4)	(3,5)
$2x+5y$	24	21	28	31

31 #

G.



$$\therefore \overline{AB} = 4\sqrt{15}, \overline{BC} = \sqrt{15}, \text{ 設 } \overline{AC} = x$$

$$\Delta ABC \text{ 面積} = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R}$$

$$S = 3\sqrt{15} + \frac{x}{2}$$

$$\Delta ABC = \sqrt{(3\sqrt{15} + \frac{x}{2})(-\sqrt{15} + \frac{x}{2})(\sqrt{15} + \frac{x}{2})(3\sqrt{15} - \frac{x}{2})} = \frac{4\sqrt{15} \cdot 2\sqrt{15} \cdot x}{4 \cdot 8}$$

$$\Rightarrow (135 - \frac{x^2}{4})(\frac{x^2}{4} - 15) = (\frac{15x}{4})^2$$

$$\Rightarrow -\frac{x^4}{16} + \frac{150}{4}x^2 - 2025 = \frac{225}{16}x^2$$

$$\Rightarrow \frac{1}{16}x^4 - \frac{375}{16}x^2 + 2025 = 0$$

$$\Rightarrow (\frac{1}{16}x^2 - 15)(x^2 - 135) = 0$$

$$\therefore x = 4\sqrt{15} \text{ or } 3\sqrt{15}$$

$$\begin{aligned}\frac{1}{16}x &= 3\sqrt{15} \\ \Rightarrow \cos C &= \frac{2^2 + 3^2 - 4^2}{2 \cdot 2 \cdot 3} < 0\end{aligned}$$

為鈍角

$$\therefore x = 4\sqrt{15} \#$$

$$H. \quad C(6,6,0), G(6,6,6) \Rightarrow P(6,6,1) > Q(0,y,0) \\ E(0,0,6), H(0,6,6) \Rightarrow R(0,3,6)$$

$\therefore P, Q, R$ 平面 \vec{AG} 不相交 \Rightarrow 平行

$$\vec{AG} = (6,6,6) \parallel (1,1,1)$$

$\Rightarrow P, Q, R$ 平面 \exists 法向量 $\vec{n} \perp (1,1,1)$

$$\vec{PQ} = (-6, y-6, -1)$$

$$\vec{PR} = (-6, -3, 5)$$

$$\therefore \vec{n} \parallel \begin{array}{cccccc} \cancel{+6} & y-6 & -1 & -6 & y-6 & \cancel{-1} \\ \cancel{+6} & -3 & 5 & -6 & -3 & \cancel{5} \end{array}$$

$$(5y-33, 36, -18+6y)$$

$$(5y-33, 36, -18+6y) \cdot (1,1,1) = 0$$

$$\Rightarrow 5y-33+36-18+6y=0 \Rightarrow 11y-15=0 \Rightarrow y = \frac{15}{11} \#$$