

1. 43659 , 數字和 $= 27 = 3^3$

$$\begin{array}{r} 161 \cancel{1} \\ 2 \overline{) 43659} \\ \underline{2} \\ 166 \\ \underline{162} \\ 45 \\ \underline{2} \\ 189 \\ \underline{189} \\ 0 \end{array}$$

$$\begin{array}{r} 14 \cancel{1} \\ 11 \overline{) 161 \cancel{1}} \\ \underline{11} \\ 51 \\ \underline{44} \\ 70 \\ \underline{77} \\ 0 \end{array}$$

$$\begin{array}{r} 21 \\ 7 \overline{) 14 \cancel{1}} \\ \underline{14} \\ 0 \\ 0 \\ 0 \end{array}$$

$$\therefore 43659 = 2 \times 11 \times 7 \times 21 \\ = 3^4 \times 7 \times 11$$

共 3 個質因數

(3) *

2. $11^3 + 12^3 + \dots + 20^3 = (1^3 + 2^3 + \dots + 20^3) - (1^3 + 2^3 + \dots + 10^3)$

$$= \left(\frac{20 \times 21}{2}\right)^2 - \left(\frac{10 \times 11}{2}\right)^2 = 210^2 - 55^2$$

$$= (210 + 55)(210 - 55) = 410 \times 5$$

(1) *

3. $\frac{1}{C_5^{39}} = \frac{C_6^{42}}{C_5^{39}} = \frac{42 \times 41 \times 40 \times 39 \times 38 \times 37}{720} = \frac{42 \times 41 \times 40}{6 \times 36 \times 35} = \frac{82}{9} \approx 9$

(4) *

4. $a = 1^n, b = 1^{13}$

$$\Rightarrow \log_9(a+b) = \log_9(1^n + 1^{13}) \approx \log_9 1^{13} = 13$$

$\because 1^n$ 比 1^{13} 小很多

(2) *

5. 設原成績為 x_i

調整後 $\Rightarrow 10\sqrt{x_i}$

$$\therefore \text{調整後標準差} = 15 = \sqrt{\frac{1}{100-1} \left[\sum_{i=1}^{100} (10\sqrt{x_i})^2 - 100 \times 65^2 \right]}$$

$$\Rightarrow 225 = \frac{1}{99} \times \left(100 \sum_{i=1}^{100} x_i - 100 \times 4225 \right)$$

$$\Rightarrow 22275 = 100 \sum_{i=1}^{100} x_i - 100 \times 4225$$

$$\Rightarrow \sum_{i=1}^{100} x_i = 4225 + 222.75 = 4447.75$$

$$\bar{x} = \frac{\sum_{i=1}^{100} x_i}{100} = 44.4775$$

(5) *

6. 若 P 在 \overleftrightarrow{AB} 上表 P.A.B 三點共線

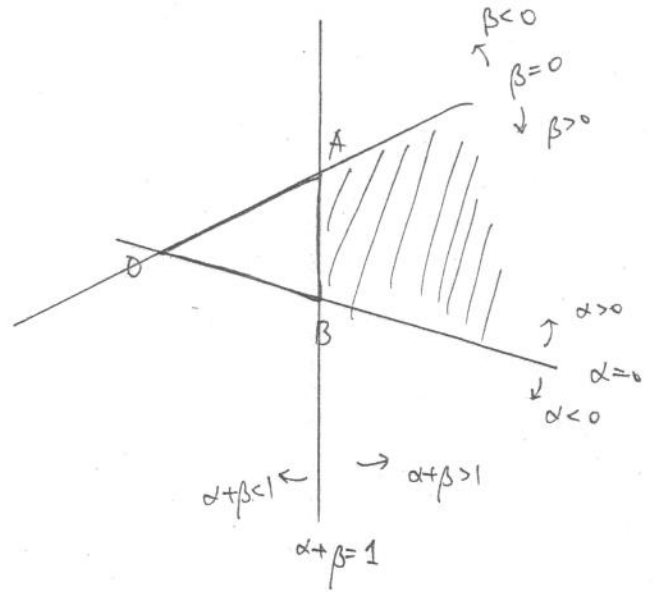
$$\vec{OP} = \alpha \vec{OA} + \beta \vec{OB} \Leftrightarrow \alpha + \beta = 1$$

若 P 在 \overleftrightarrow{OA} 上

$$\vec{OP} = \alpha \vec{OA} + \beta \vec{OB} \Leftrightarrow \beta = 0$$

若 P 在 \overleftrightarrow{OB} 上

$$\vec{OP} = \alpha \vec{OA} + \beta \vec{OB} \Leftrightarrow \alpha = 0$$



灰色區域表示 $\alpha + \beta > 1, \alpha > 0, \beta > 0$.

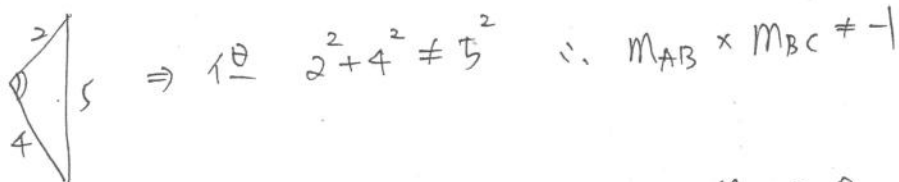
(1)(2) *

7. (1) m_{CD} 最斜且斜率為正 $\Rightarrow m_{CD}$ 最大

(2) m_{BC} 最斜且斜率為負 $\Rightarrow m_{BC}$ 最小.

B' m_{BC}, m_{CD} 一樣斜且一正一負 $\Rightarrow m_{BC} = -m_{CD}$

(4) 若 $m_{AB} \times m_{BC} = -1$. 表 $\overleftrightarrow{AB}, \overleftrightarrow{BC}$ 互相垂直



(5) $m_{CD} > m_{AB} > 0$ 且 $m_{DA} = -m_{AB} \Rightarrow m_{CD} + m_{DA} > 0$

(2)(3)(5)

8. 三平面相交情形有 8 種, 解分別為

- ① 無解
- ② 無限多解
- ③ 一解

∵ 已經知道交於 2 點 \Rightarrow 無限多解

又三平面互異, 且無限多解 \Rightarrow 三平面交一線

\therefore 此交線通過 $(-1, 2, 0)$ 及 $(3, 0, 2) \Rightarrow \vec{L} \parallel (4, -2, 2) \parallel (2, -1, 1)$
(L)

Δ 可表成 $\frac{x+1}{2} = \frac{y-2}{-1} = \frac{z}{1}$

將每個選項代入 Δ , 發現只有 (2) 符合

(2) #

9. 已知 $0 < \theta < \frac{\pi}{4}$, 此時

(1) $\sin \theta$ 遞增, $\cos \theta$ 遞減 且 $\sin 45^\circ = \cos 45^\circ \therefore \sin \theta < \cos \theta$

(2) $\tan 45^\circ = 1, \sin 45^\circ = \frac{\sqrt{2}}{2}$, 當 θ 很接近 45° 時 $\tan \theta > \frac{\sqrt{2}}{2} > \sin \theta$

(3) $\tan 45^\circ = 1, \cos 45^\circ = \frac{\sqrt{2}}{2}$, 當 θ 很接近 45° 時, $\tan \theta > \cos \theta$

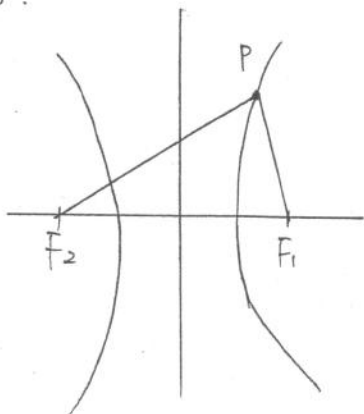
(4) 當 $\theta = 22.5^\circ$ 時, $\sin 2\theta = \cos 2\theta$

(5) $\frac{1}{2} \tan \theta = \frac{1}{2} \times \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \frac{\tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} < \tan \frac{\theta}{2}$ (令 θ 在 $0, 1$ 之間)

$\left(\begin{array}{l} \because \tan 45^\circ = 1 \text{ 且 } \tan 0 = 0, \text{ 又 } 0 < \frac{\theta}{2} < \frac{\pi}{8} < \frac{\pi}{4} \\ \therefore 0 < \tan^2 \frac{\theta}{2} < 1 \Rightarrow 0 < 1 - \tan^2 \frac{\theta}{2} < 1 \end{array} \right)$

(1) (5) #

10.



$\overline{F_1 F_2} = 2c = 10$

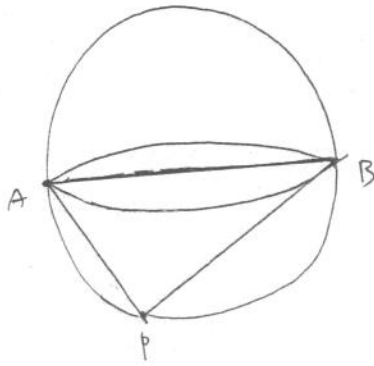
雙曲線定義 $|\overline{PF_1} - \overline{PF_2}| = 2a = 6$

\therefore 等號 $\therefore \overline{PF_2} = \overline{F_1 F_2} = 10 \Rightarrow \overline{PF_1} = 4 \text{ or } 16$

$\therefore \Delta PF_1 F_2$ 之周長為 $= 4 \text{ or } 36$

(2) (5) #

11.



若 P 在球上 $\Rightarrow \overline{PA}^2 + \overline{PB}^2 = \overline{AB}^2 = 100$

[法一] $(\overline{PA}^2 + \overline{PB}^2)(1^2 + 1^2) \geq (\overline{PA} + \overline{PB})^2$

$\Rightarrow \overline{PA} + \overline{PB} \leq 10\sqrt{2} = 14.14 \dots$

[法二] P 在正中間時, $\overline{PA} + \overline{PB}$ 最大 = $10\sqrt{2}$.
(即 $\overline{PA} = \overline{PB}$)

$\Rightarrow \overline{PA} + \overline{PB} \leq 10\sqrt{2} = 14.14 \dots$

$\therefore \overline{PA} + \overline{PB} = 14$

\therefore 可能在球上, 球外, 球內

(3)(4)(5) 皆對.

若 P 在 \overline{AB} 上 $\Rightarrow \overline{PA} + \overline{PB} = \overline{AB} = 10$ 不可能

P 可能在 \overleftrightarrow{AB} 上但由 (1) 知 P 不在 \overline{AB} 上

(2)(3)(4)(5) #

填充題

A.

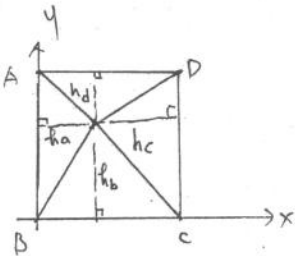
$$\begin{array}{r} 1 \ 0 \ -1 \ 4 \\ 1 \ 1 \ 1 \ p = 8 \\ 1 \ 1 \ 2 \end{array} \quad \begin{array}{r} -1 \ p \ 2 \\ -1 \ -1 \ -2 \\ \hline p+1 \ 4 \ 8 \\ 4 \ 4 \ 8 \\ \hline 0 \end{array}$$

$\therefore \begin{cases} p+1=4 \\ 8=8 \end{cases}$

$\Rightarrow \begin{cases} p=3 \\ 8=8 \end{cases}$

3; 8 #

B.



$\therefore \triangle PDA \sim \triangle PBC = 1:2 \Rightarrow h_d = h_b = 1:2$

$\therefore \triangle PAB \sim \triangle PCD = 2:3 \Rightarrow h_a = h_c = 2:3$

\therefore P 點之 y 坐標 $\frac{2}{3}$; x 坐標 $\frac{2}{5}$

$(\frac{2}{5}, \frac{2}{3})$ #

C. 設向前是步 $x=k \Rightarrow$ 向後是步 $6-x=k$

$x + 6 - x = 4 \Rightarrow x = 5$

\therefore 向前 5 次, 向後 1 次. 即 $\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \leftarrow$, 三個排列 $\frac{6!}{5!1!} = 6$

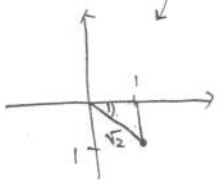
6 #

$$D. 1+z+\dots+z^9 = \frac{(1-z^{10})}{1-z} = \frac{1-(1-i)^{10}}{1-(1-i)}$$

必看到複數很多=2方 \Rightarrow 化極式, 利用棣美弗公式展開

$$(1-i)^{10} = (\sqrt{2}(\cos(-45^\circ) + i\sin(-45^\circ)))^{10} = 32[\cos(-450^\circ) + i\sin(-450^\circ)]$$

$$= 32(-i) = -32i$$

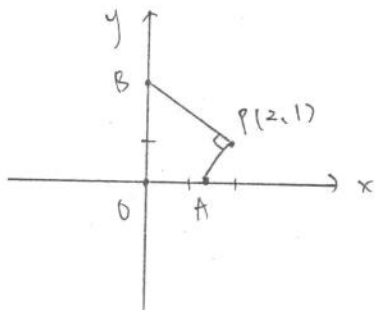


$$\text{原式} = \frac{1+32i}{i} = -i+32$$

$$\therefore a=32, b=-1$$

32; -1 #

E. \because A 在 x 軸上, 設 A(a, 0)
B 在 y 軸上, 設 B(0, b)



$$\because \vec{AP} \perp \vec{BP}$$

$$\therefore (2-a, 1) \cdot (2, 1-b) = 0$$

$$\Rightarrow 4-2a+1-b=0 \Rightarrow 2a+b=5$$

求 $\triangle OAB$ 面積 = $\frac{1}{2}ab$ 最大.

$$\Rightarrow \frac{1}{2}ab = \frac{1}{2}a(5-2a)$$

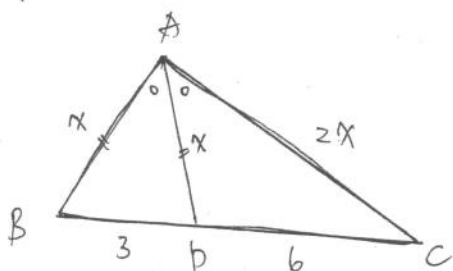
$$= -a^2 + \frac{5}{2}a$$

$$= -(a - \frac{5}{4})^2 + \frac{25}{16}$$

\therefore 最大面積 $\frac{25}{16}$

$\frac{25}{16}$ #

F.



$$\text{設 } \overline{AB} = \overline{AD} = x$$

\because AD 為角平分線

$$\therefore \frac{\overline{AB}}{\overline{AC}} = \frac{\overline{DB}}{\overline{DC}} \Rightarrow \overline{AC} = 2x$$

$$\cos B = \frac{x^2+3^2-x^2}{2 \cdot x \cdot 3} = \frac{x^2+9^2-(2x)^2}{2 \cdot 9 \cdot x}$$

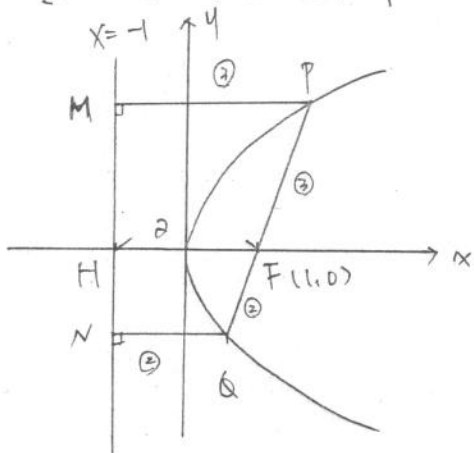
($\triangle ABD$) ($\triangle ABC$)

$$\Rightarrow 3(9) = 81 - 2x^2 \Rightarrow x^2 = 18 \Rightarrow x = 3\sqrt{2} \text{ (取正)}$$

$$\cos \angle BAD = \frac{(3\sqrt{2})^2 + (3\sqrt{2})^2 - 3^2}{2 \cdot 3\sqrt{2} \cdot 3\sqrt{2}} = \frac{27}{36} = \frac{3}{4}$$

$\frac{3}{4}$ #

G. 必看到圓錐曲線常要用定義



準線

$$\overline{PF} = \overline{PM}, \quad \overline{FO} = \overline{ON}$$

$$\therefore \overline{PM} = \overline{ON} = 3 = 2 + \overline{MH} \quad \overline{MH} = \overline{HN} = 3 = 2 + \overline{FN}$$

$$\text{設 } \overline{PM} = 3t, \quad \overline{PN} = 2t$$

$$\overline{FH} = \frac{3 \times \overline{ON} + 2 \times \overline{PM}}{3+2} = \frac{12t}{5} = 2 \Rightarrow t = \frac{5}{6}$$

$$\therefore \overline{PM} = \frac{5}{2} \Rightarrow P \text{ 之 } x \text{ 坐標 } \frac{3}{2}$$

$\frac{3}{2}$ #

H.

$$3^{18} = 1 \cdot 3^{18}$$

$$= 3 \cdot 3^{17}$$

$$= 9 \cdot 3^{16}$$

$$= 27 \cdot 3^{15}$$

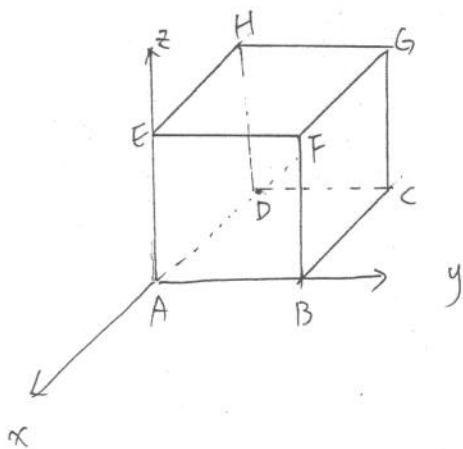
在此之間 $\Rightarrow 15 < k < 16$

$$\Rightarrow k = 15$$

15 #

I.

※此類型可以坐標坐, 即把圖形坐標化後再解會變得容易許多。



$$A(0,0,0), \quad \overrightarrow{AB} = (0,1,0)$$

$$\overrightarrow{AD} = (-1,0,0)$$

$$\overrightarrow{AE} = (0,0,1)$$

$$\overrightarrow{AP} = \left(\frac{1}{2}, \frac{3}{4}, \frac{2}{3}\right), \quad \therefore P \text{ 之坐標 } \left(\frac{1}{2}, \frac{3}{4}, \frac{2}{3}\right)$$

$$\overrightarrow{AB} = (0, t, 0) \quad t \in \mathbb{R}$$

$$\text{P 到 } \overleftrightarrow{AB} \text{ 距離} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(t - \frac{3}{4}\right)^2 + \left(\frac{2}{3}\right)^2}$$

$$= \sqrt{\left(t - \frac{3}{4}\right)^2 + \frac{1}{4} + \frac{4}{9}} = \sqrt{\left(t - \frac{3}{4}\right)^2 + \frac{25}{36}}$$

$$\therefore \sqrt{\frac{25}{36}} = \frac{5}{6}$$

$\frac{5}{6}$ #