

$$1. \begin{cases} a > 1 : x > y \Leftrightarrow a^x > a^y \\ 0 < a < 1 : x > y \Leftrightarrow a^x < a^y \end{cases}$$

$$x^2 + \frac{2}{3} > \frac{2}{3}$$

$$\Rightarrow 2) x^2 + \frac{2}{3} > 2) \frac{2}{3} = 3^2 = 9$$

(3) *

2.

$$ERA = \frac{E}{n} \times 9$$

$$90 \text{局} \Rightarrow 3.2 = \frac{E}{90} \times 9 \Rightarrow E = 3.2$$

$$\text{再6局} \Rightarrow ERA = \frac{3.2}{96} \times 9 = \frac{1}{3} \times 9 = 3$$

(2) *

3.

$$\text{相同時間, 甲, 乙所走的距離比} \Rightarrow \frac{\text{甲}_{\text{走}}}{\text{乙}_{\text{走}}} = \frac{30 \times 1}{50 \times 2} = \frac{3}{10}$$

$$\text{現在甲走 45 公尺} \Rightarrow \text{乙走 } 45 \times \frac{10}{3} = 150 \text{ 公尺}$$

(4) *

4.

A □ - □ □ □ □

↑ ↑

1 25
(不能0)

$$(10 \times 10 \times 10 - 10 \times 1 \times 1)$$

(任意 - 後兩碼均為4)

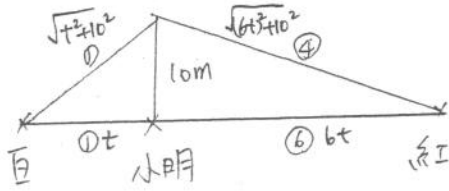
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淨 = 碼是 4.

$$\Rightarrow \frac{1}{x} \times 25 \times (1000 - 10) = 25 \times 990$$

(4) *

5.



設小明與巨遠距離 t 公尺

$$\Rightarrow 4 \sqrt{t^2 + 10^2} = \sqrt{(6t)^2 + 10^2}$$

$$\Rightarrow 16(t^2 + 100) = 36t^2 + 100$$

$$\Rightarrow 16t^2 + 1600 = 36t^2 + 100$$

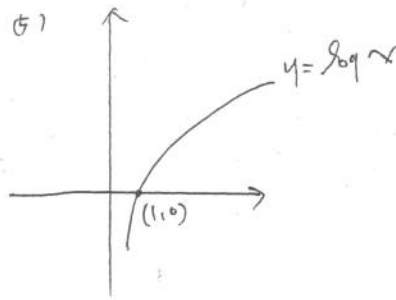
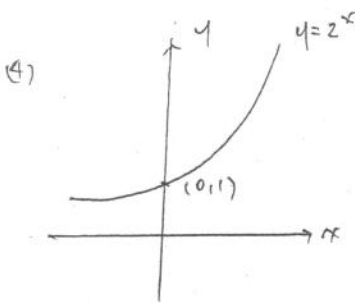
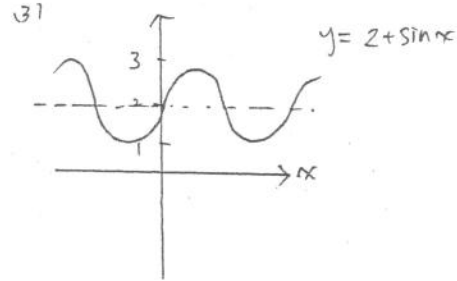
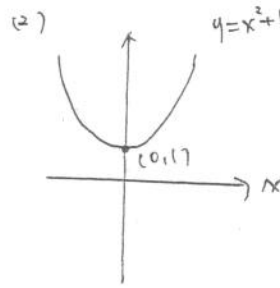
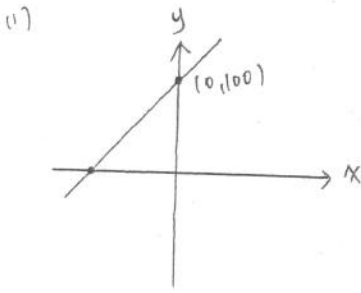
$$\Rightarrow 20t^2 = 1500$$

$$\Rightarrow t^2 = 75 \Rightarrow t = 5\sqrt{3}$$

∴ 巨兩隻距離 = $7t = 35\sqrt{3} \approx 60.62$

(1) #

6.



(2)(3)(4) #

7.

(1) 有可能 80 人，落在 2 班。(X)

(2) 有可能 80 人都是女生 (X)

(3) 小文抽中機率 = $\frac{80}{800} = \frac{1}{10}$ (X)

小美抽中機率 = $\frac{80}{800} = \frac{1}{10}$

(4) 甲乙同時抽中 = $\frac{80}{800} \times \frac{79}{799}$ (0)

甲丙同時抽中 = $\frac{80}{800} \times \frac{79}{799}$

(5) AB 同時抽中 = $\frac{80}{800} \times \frac{79}{799} < \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ (0)

(4)(5) #

8.

1) a_1, a_2, a_3 成等差. 可設三數為 $a, a+d, a+2d$

$$\begin{aligned} \because a_1 < a_2 &\Rightarrow d > 0 & (x) \\ a_2 > a_3 &\Rightarrow d < 0 & (x) \end{aligned}$$

2) $\{x\}: b_1 = -1, b_2 = 2, b_3 = -4$ (o)

$$\Rightarrow -1, 2, -4 \text{ 成等比且 } -1 < 2, 2 > -4$$

3) 反例: a_1, a_2, a_3 分別為 $-2, 1, 4$ 成等差 (x)

$$\Rightarrow a_1 + a_2 < 0, \text{ 但 } a_2 + a_3 > 0$$

4) b_1, b_2, b_3 成等比. 可設三數為 b, br, br^2 (o)

$$\because b_1 \cdot b_2 < 0 \Rightarrow b \cdot br < 0 \Rightarrow r < 0$$

$$\therefore b_2 \cdot b_3 = (br)(br^2) = \underbrace{b^2}_{>0} \cdot \underbrace{r^2}_{>0} \cdot \underbrace{r}_{<0} < 0$$

5) 反例: b_1, b_2, b_3 分別為 $4, 6, 9$ 成等比且皆為正整數 (x)

$$\Rightarrow 4^0 \cdot 4 \neq 6$$

(2)(4) #

9.

$$n_A \times n_B = 10^{10} \Rightarrow 1 \leq n_A \leq 10^{10}$$

1) $\log 1 \leq \log n_A \leq \log 10^{10} \Rightarrow 0 \leq P_A \leq 10$ (x)

2) $P_A = 5 \Rightarrow \log n_A = 5 \Rightarrow n_A = 10^5 \Rightarrow n_B = \frac{10^{10}}{10^5} = 10^5$

$$\therefore n_A = n_B \quad (o)$$

3) $P_A = 4 \Rightarrow \log n_A = 4 \Rightarrow n_A = 10^4$
 $P_{A'} = 8 \Rightarrow \log n_{A'} = 8 \Rightarrow n_{A'} = 10^8 \Rightarrow \frac{n_{A'}}{n_A} = \frac{10^8}{10^4} = 10^4 \neq 2$ (x)

4) 設昨天 $P_A = k$, 則今天 $P_{A'} = k+1$

$$\Rightarrow k = \log n_A \Rightarrow n_A = 10^k \Rightarrow 10^{k+1} - 10^k \neq 10 \quad (x)$$

$$(k+1 = \log n_{A'} \Rightarrow n_{A'} = 10^{k+1})$$

$$(5) \quad n_B = 5 \times 10^4 \Rightarrow n_A = \frac{10^{10}}{5 \times 10^4} = 2 \times 10^5$$

$$p_A = \log n_A = \log (2 \times 10^5) = 5 + \log 2 = 5.3010 \quad (0)$$

(5) *

10. 求公因式 \Rightarrow 一個有未知數, 一個沒有未知數.
 \Rightarrow 將沒有未知數者因式分解.

$$g(x) = x^3 + x^2 - 2 = (x-1)(x^2 - 2x + 2)$$

實數不可再分解 ($D < 0$)

\therefore 有次數大於 0 之公因式 \Rightarrow 最高公因式可能

- ① $x-1$
- ② $x^2 - 2x + 2$
- ③ $(x-1)(x^2 - 2x + 2)$

1) $g(x)=0$ 的根 $\Rightarrow x=1, \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm 2i \Rightarrow$ 恰有一實根 (0)

2) 若最高公因式為 $x^2 - 2x + 2 \Rightarrow f(x)$ 不一定有實根. (X)
 $\text{Ex: } f(x) = x^2 - 2x + 2$

3) $g(x)=0$ 只有一實根 1, $\therefore f(x)=0, g(x)=0$ 有共同實根 \Rightarrow 此根為 1. (0)

4) $f(x)=0, g(x)=0$, 有共同實根 \Rightarrow 可能之最高公因式 $x-1$
 $(x-1)(x^2 - 2x + 2)$

\Rightarrow 不一定是二次 (X)

5) $f(x)=0, g(x)=0$ 沒有共同實根 \Rightarrow 最高公因式 = 1 \Rightarrow 一次 (0)

(1)(3)(5) *

11.

1) L_1, L_2 均過點 $(1, -3, -4) \Rightarrow L_1, L_2$ 相交 (0)

2) $\vec{L}_2 = (1, 3, 4) \quad \vec{L}_3 = (1, 3, 4) \Rightarrow \vec{L}_2 \parallel \vec{L}_3$

$\times (0, -3, -4)$ 不在 L_3 上 $(\frac{0}{1} \neq \frac{-3}{3} = \frac{-4}{4}) \Rightarrow L_2, L_3$ 不重合

$\Rightarrow L_2 \parallel L_3$ (0)

3) 若 \overline{PQ} 是 P 到 L_3 之最短距離

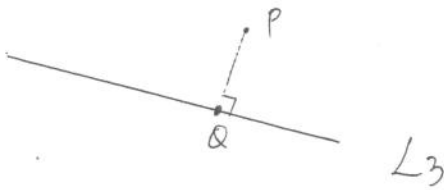
$$\vec{PQ} = (0, 3, 4)$$

$$\Rightarrow \overline{PQ} \perp L_3$$

$$\vec{L}_3 = (1, 3, 4)$$

$$\vec{PQ} \cdot \vec{L}_3 = 0 + 9 + 16 = 25 \neq 0$$

$$\therefore \overline{PQ} \not\perp L_3 \quad (X)$$



4)

$$\begin{cases} x=0 \\ \frac{y+3}{4} = \frac{z+4}{-3} \Rightarrow -3y-9 = 4z+16 \Rightarrow 3y+4z+25=0 \end{cases}$$

$$\Rightarrow \begin{cases} x=0 \\ 3y+4z=25 \end{cases}$$

兩面式 $\Rightarrow \vec{n} \perp (1, 0, 0)$
 $\vec{n} \perp (0, 3, 4)$

$$\begin{array}{r} \begin{array}{cccc|c} \cancel{6} & 0 & 1 & 0 & 0 \\ 0 & \cancel{3} & \cancel{4} & 0 & \cancel{3} & 4 \end{array} \\ \hline 0 & -4 & 3 & & \end{array}$$

$$\Rightarrow \vec{n} \parallel (0, -4, 3)$$

$$\vec{n} \cdot \vec{L}_1 = (0, -4, 3) \cdot (1, 6, 8) = -24 + 24 = 0 \Rightarrow \vec{n} \perp \vec{L}_1$$

$$\vec{n} \cdot \vec{L}_2 = (0, -4, 3) \cdot (1, 3, 4) = -12 + 12 = 0 \Rightarrow \vec{n} \perp \vec{L}_2$$

(O)

5) L_1 與 L_2 相交且 L_2 與 L_3 平行

\Rightarrow 看 L_1, L_3 的關係. $(\vec{L}_1 \times \vec{L}_3)$

看 L_1, L_3 是否相交, 設 L_1, L_3 之交點 P .

$$\because P \text{ 在 } L_1 \text{ 上} \Rightarrow \text{設 } P(t, -3+6t, -4+8t)$$

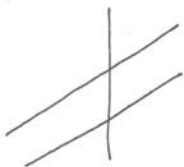
$$\because P \text{ 在 } L_2 \text{ 上} \Rightarrow \text{設 } P(s, 3s, 4t)$$

$$\Rightarrow \begin{cases} t = s & \text{--- ①} \\ -3+6t = 3s & \text{--- ②} \\ -4+8t = 4s & \text{--- ③} \end{cases} \quad \text{①代②} \Rightarrow -3+6t=3t \Rightarrow t=1 \Rightarrow s=1$$

$$t=1, s=1 \text{ 代回 ③ 檢驗 } -4+8=4. \quad (\checkmark)$$

即 L_1 與 L_3 交於 $P(1, 3, 4)$

$\therefore \Rightarrow L_1, L_2, L_3$ 共平面 (O)



(1)(2)(4)(5) \neq

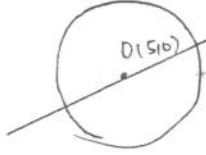
12. $x^2 + y^2 - 10x + 9 = 0$

$\Rightarrow (x-5)^2 + y^2 = -9 + 25 \Rightarrow (x-5)^2 + y^2 = 4^2$

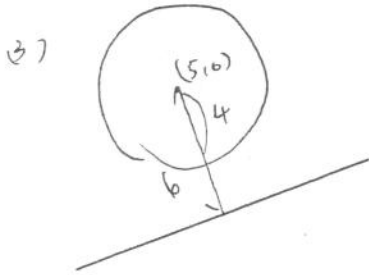
1) 圓心 $(5, 0)$ (0)

2) $3x + 4y - 15 = 0$

$d(O, L) = \frac{|15 + 0 - 15|}{\sqrt{3^2 + 4^2}} = 0$

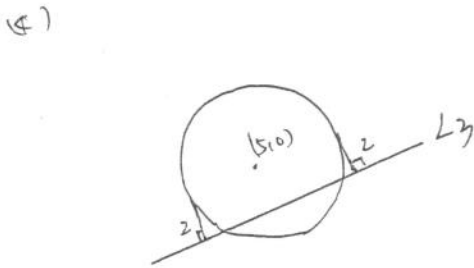


\therefore 最遠距離 = 半徑 = 4 (0)



$d(O, L) = \frac{|15 + 0 + 15|}{\sqrt{3^2 + 4^2}} = 6$

\Rightarrow 不相切 (相離) (x)

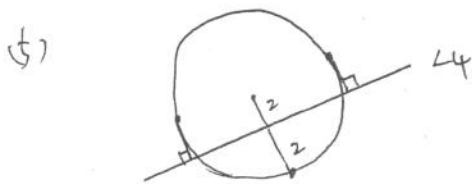


$d(O, L_2) = \frac{|15 + 0|}{\sqrt{3^2 + 4^2}} = 3$

\Rightarrow L_3 右下半部 最遠距離 = $4 - 3 = 1$

L_3 左上半部 最遠距離 = $4 + 3 = 7$

\Rightarrow 有 2 個真距離 2 (0)
對稱



$d(O, L_3) = \frac{|15 + 0 - 5|}{\sqrt{3^2 + 4^2}} = 2$

\Rightarrow L_4 右下半部 有 1 個距離 2

L_4 左上半部 有 2 個距離 2

\Rightarrow 共 3 個真距離 2 (x)

(1)(2)(4) #

A. 設 $D(x, y, z)$

$$\vec{DA} = (-1-x, 6-y, -z)$$

$$\vec{DB} = (3-x, -1-y, -2-z)$$

$$\vec{DC} = (4-x, 4-y, 5-z)$$

$$3\vec{DA} - 4\vec{DB} + 2\vec{DC} = \vec{0}$$

$$\Rightarrow 3(-1-x, 6-y, -z) - 4(3-x, -1-y, -2-z) + 2(4-x, 4-y, 5-z) = (0, 0, 0)$$

$$\Rightarrow \begin{cases} -3-3x-12+4x+8-2x=0 & \Rightarrow -x-7=0 & \Rightarrow x=-7 \\ 18-3y+4+4y+8-2y=0 & \Rightarrow -y+30=0 & \Rightarrow y=30 \\ -3z+8+4z+10-2z=0 & \Rightarrow -z+18=0 & \Rightarrow z=18 \end{cases}$$

$$\underline{(-7, 30, 18)}^*$$

B.

A 在直線 $3x-y=0$ 上 \Rightarrow 設 $A(t, 3t)$ B 在 x 軸上 \Rightarrow 設 $B(b, 0)$

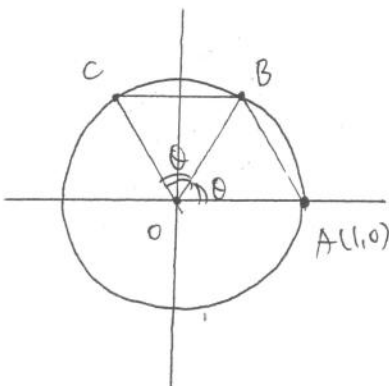
$$A, B \text{ 中點} = \frac{A+B}{2} = \left(\frac{t+b}{2}, \frac{3t}{2} \right) = \left(\frac{7}{2}, 6 \right)$$

$$\Rightarrow \begin{cases} \frac{t+b}{2} = \frac{7}{2} & \Rightarrow b=3 \\ \frac{3t}{2} = 6 & \Rightarrow t=4 \end{cases}$$

$$A(4, 12)$$

$$\underline{B(3, 0)}^*$$

C.



$$\because \overline{AB} = \overline{BC}$$

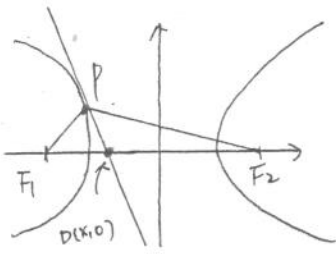
$$\Rightarrow \angle AOB = \angle BOC = \theta$$

$$\because \Delta AOB \text{ 面積} = \frac{1}{2} \cdot 1 \cdot 1 \cdot \sin \theta = \frac{3}{10} \Rightarrow \sin \theta = \frac{3}{5} \Rightarrow \cos \theta = \frac{4}{5}$$

$$\Rightarrow \Delta AOC \text{ 面積} = \frac{1}{2} \cdot 1 \cdot 1 \cdot \sin 2\theta = \frac{1}{2} \cdot 2 \cdot \sin \theta \cdot \cos \theta = \frac{3}{5} \cdot \frac{4}{5} = \frac{12}{25}$$

$$\underline{\frac{12}{25}}^*$$

D. 橢圓、雙曲線之光學性質 $\Rightarrow \angle F_1 P F_2$ 之角平分線 與 過 P 之切線有關



雙曲線: $\angle F_1 P F_2$ 之角平分線 即 切線.

$\therefore (-4, 1)$ 在曲線上, 求切線 \Rightarrow 一半公式

$$\left(\begin{array}{l} x^2 \rightarrow x_0 x \\ y^2 \rightarrow y_0 y \end{array} \quad \begin{array}{l} x \rightarrow \frac{x_0 + x}{2} \\ y \rightarrow \frac{y_0 + y}{2} \end{array} \right)$$

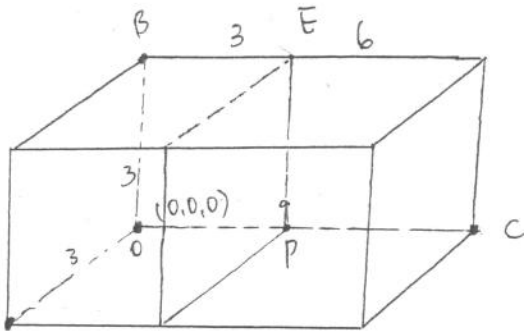
$$\therefore x^2 \rightarrow -4x, \quad y^2 \rightarrow 1 \cdot y$$

$$\frac{x^2}{8} - y^2 = 1 \xrightarrow{\text{一半公式}} \frac{-4x}{8} - y = 1 \Rightarrow -x - 2y = 2$$

$$D(x, 0) \Rightarrow -x - 0 = 2 \Rightarrow x = -2$$

-2 #

E.



$$A(2, 2, 1) \Rightarrow \overline{OA} = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$B(2, 4, 2) \Rightarrow \overline{OB} = \sqrt{2^2 + 4^2 + 2^2} = 6$$

$$C(3, -6, 6) \Rightarrow \overline{OC} = \sqrt{3^2 + (-6)^2 + 6^2} = 9$$

A

\therefore 要切出正立方體 \Rightarrow 切平面 E 之法向量 $\vec{n} \parallel \overline{OC} = (3, -6, 6) \parallel (1, -2, 2)$

找點 P, P 在 \overline{BC} 上 使得 $\overline{OP} = 3, \overline{PC} = 6$

$$P = \frac{1 \cdot C + 2 \cdot O}{1+2} = \frac{(3, -6, 6) + 2(0, 0, 0)}{3} = (1, -2, 2)$$

$$E: x - 2y + 2z = \frac{(1, -2, 2) \cdot (3, -6, 6)}{9} \cdot 9$$

$$(b, c, d) = (-2, 2, 9) \#$$

F. $b^2 = 9a \Rightarrow a = \frac{b^2}{9}$

$$a + 2b > 2f_0 \Rightarrow \frac{b^2}{9} + 2b > 2f_0 \Rightarrow b^2 + 18b > 2520 \Rightarrow (b+9)^2 > 2601 = 51^2$$

$$\Rightarrow b+9 > 51 \text{ or } b+9 < -51 \Rightarrow b > 42 \text{ or } b < -60 \quad (\text{取正}) \Rightarrow b > 42$$

$\because a = \frac{b^2}{9} \therefore b \text{ 愈小, } a \text{ 愈小}$

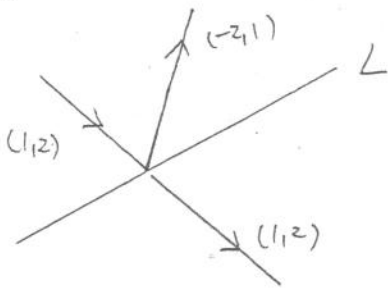
$\Rightarrow b=43 \Rightarrow a = \frac{43^2}{9}$ (不是整數, 不合)

$b=44 \Rightarrow a = \frac{44^2}{9}$ (不是整數, 不合)

$b=45 \Rightarrow a = \frac{45^2}{9} = 225$

225 #

9.



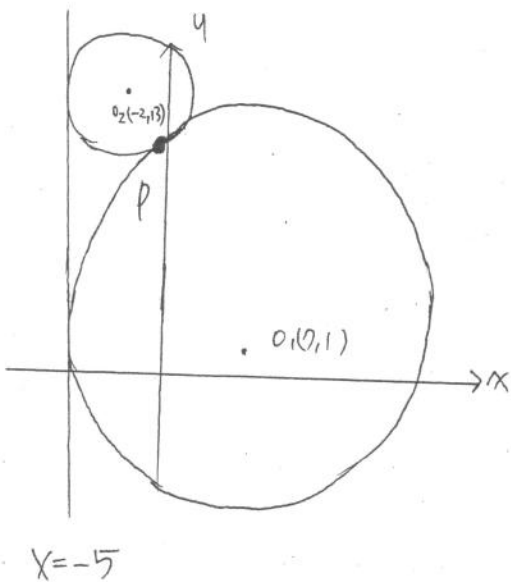
如圖二為 $(-2, 1)$ 和 $(1, 2)$ 之角平分線方向。

又 $|(-2, 1)| = \sqrt{3}, \quad |(1, 2)| = \sqrt{3}$

$\therefore \vec{\Delta} = (-2, 1) + (1, 2) = (-1, 3) \parallel (1, -3)$

-3 #

11. 圓錐曲線 \Rightarrow 常考定義式



O_1 圓心 $(0, 1) \Rightarrow$ 連心距 $= \sqrt{9 + (-1)^2} = 10$

O_2 圓心 $(-2, 3) \Rightarrow$ 半徑和 $= 1 + 3 = 4$

\Rightarrow 外切, 如左圖。

設兩圓切點 P

$\Rightarrow PO_1 = d(O_1, L) = 10$

$PO_2 = d(O_2, L) = 3$

$\Rightarrow P$ 為焦點, L 為準線之拋物線。

$\Rightarrow P = \frac{1 \cdot O_1 + 4O_2}{1+4}$

$\Rightarrow P = \frac{1 \cdot (0, 1) + 4 \cdot (-2, 3)}{5} = \left(\frac{-1}{5}, \frac{5}{5} \right) = \left(\frac{-1}{5}, 1 \right)$ #