

1. $a_1 + 2 + (a_2 + 4) + \dots + (a_k + 2k) + \dots + (a_{10} + 20) = 240$

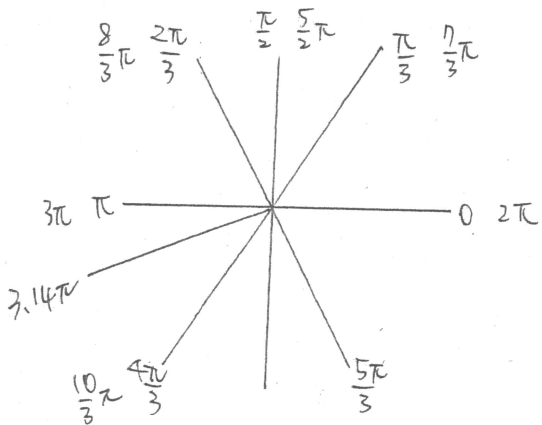
$$\Rightarrow (a_1 + a_2 + \dots + a_{10}) + (2 + 4 + \dots + 20) = 240$$

$$\Rightarrow (a_1 + a_2 + \dots + a_{10}) + \frac{10(2+20)}{2} = 240$$

$$\Rightarrow a_1 + a_2 + \dots + a_{10} + 110 = 240 \Rightarrow a_1 + a_2 + \dots + a_{10} = 130$$

(3) #

2. $\frac{2\pi}{3}$ 與 $\frac{4\pi}{3}$ 有關 $\Rightarrow \cos \frac{\pi}{3} = \frac{1}{2}$



$$\cos \pi^2 \approx \cos (3.14\pi)$$

$$3\pi \leq 3.14\pi \leq \frac{10}{3}\pi$$

$$\therefore \cos 3\pi = \cos \pi = -1$$

$$\cos \frac{10}{3}\pi = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$\therefore -1 < a < -\frac{1}{2}$$

(2) #

3. 看到除法 \Rightarrow 除法原理 (被除式 = 除式 \times 商 + 餘式)

$$f(x) = g(x) \cdot Q(x) + (x^4 - 1)$$

$$\Rightarrow (f(x), g(x)) = (g(x), (x^4 - 1))$$

\therefore 可能之公因式 \Rightarrow 必定是 $(x^4 - 1)$ 的因式

\Rightarrow 僅 $(x^3 - 1)$ 不是 $x^4 - 1$ 之因式

* 公因式可以有係數倍。

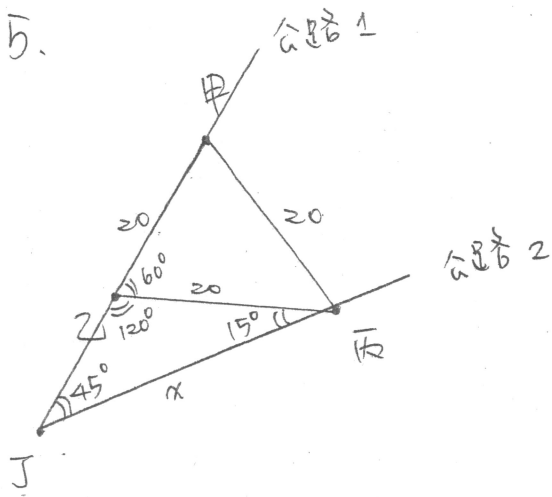
(4) #

4. 同甲 同乙 同丙

$$\frac{C_2^3 + C_2^4 + C_2^5}{C_2^{12}} = \frac{3 + 6 + 10}{66} = \frac{19}{66} \approx 0.29$$

(5) #

5.



設丙丁兩鎮距 x 公里

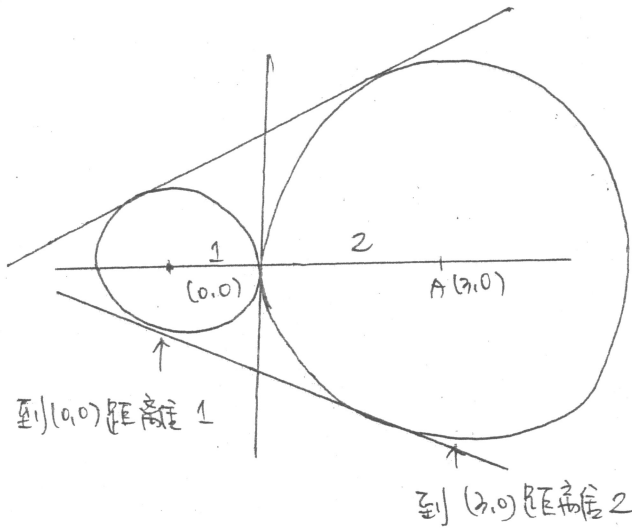
$$\Rightarrow \frac{x}{\sin 120^\circ} = \frac{20}{\sin 45^\circ}$$

$$\Rightarrow \frac{x}{\frac{\sqrt{3}}{2}} = \frac{20}{\frac{\sqrt{2}}{2}}$$

$$\Rightarrow x = 20 \times \frac{2}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = 10\sqrt{6} \approx 24.49$$

(1) *

6.



圓心到直線的距離即切線

∵ 滿足此條件 ⇒ 即公切線

⇒ 公切線有 3 條

(3) *

7. 可化分數的數稱有理數.

1) 3.1416 (0)

2) $\sqrt{3}$ (x)

3) $\log_{10} \sqrt{5} + \log_{10} \sqrt{2} = \log_{10} \sqrt{10} = \log_{10} (10^{\frac{1}{2}}) = \frac{1}{2}$ (0)

4) $\frac{\sin 15^\circ}{\cos 15^\circ} + \frac{\cos 15^\circ}{\sin 15^\circ} = \frac{\sin^2 15^\circ + \cos^2 15^\circ}{\sin 15^\circ \cos 15^\circ} = \frac{1}{\frac{1}{2} \sin 30^\circ} = \frac{1}{\frac{1}{2} \times \frac{1}{2}} = 4$ (0)

5) 若 $x^3 - 2x^2 + x - 1$ 有有理根 ⇒ 必是 ± 1 .

① 檢驗 1 ⇒ $1^3 - 2 + 1 - 1 = -1 \neq 0$.

② 檢驗 -1 ⇒ $(-1)^3 - 2(-1)^2 - 1 - 1 = -5 \neq 0$

∴ 無有理根 (x)

(1)(3)(4) *

8.

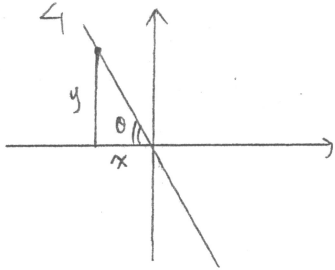
$$1) m_3 > 0 > m_1 > m_2 \quad (x)$$

$$2) \angle_1 \perp \angle_3 \Rightarrow m_1 \cdot m_3 = -1$$

$$\Rightarrow m_1 \cdot m_4 = -1 \quad (o)$$

$$\angle_3 \parallel \angle_4 \Rightarrow m_3 = m_4$$

3)



可量_量見_見知 $y > x$ (當 $y = x \Rightarrow m = -1$)

$$\Rightarrow m < -1 \quad (+, - \text{表方向, 愈斜數字愈大}) \quad (o)$$

$$4) m_1 \cdot m_3 = -1$$

$$\text{又 } m_2 < m_1 < 0 \Rightarrow \frac{m_2}{m_1} > 1$$

同乘負號

$$\Rightarrow \frac{m_2}{m_1} \cdot (m_1 \cdot m_3) < 1 \cdot (-1)$$

$$\Rightarrow m_2 \cdot m_3 < -1 \quad (o)$$

$$5) \angle_4 \text{ 與 } y \text{ 軸之交點 } (0, c) \Rightarrow \text{在 } x \text{ 軸下方} \Rightarrow c < 0 \quad (x)$$

(2)(3)(4) *

9. 設甲地參訪者聽過該產品比例 \hat{p}_1 , 參訪人數 $n_1 \Rightarrow$ 信賴區間 $[\hat{p}_1 - 2\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}}, \hat{p}_1 + 2\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}}]$
 乙地 \hat{p}_2 n_2

$$1) \hat{p}_1 = \frac{0.50 + 0.58}{2} = 0.54 \quad (o)$$

$$2) 0.04 = 2\sqrt{\frac{0.54 \times 0.46}{n_1}} \Rightarrow 0.54 \times 0.46 > 0.12 \times 0.88$$

$$0.04 = 2\sqrt{\frac{0.12 \times 0.88}{n_2}} \therefore n_1 > n_2 \quad (o)$$

3) 解讀應為: 相同調查做 100 次, 約有 95 次算出來之信賴區間含真值 (x)

4) 解讀應同上 (x)

5) 重新調查 $n \rightarrow 4n$ (人數 4 倍), 但 \hat{p} 值會改變

\Rightarrow 無法確認信賴區間 (x)

(1)(2) *

10.

$$\begin{cases} x+2y+az=1 & \text{--- ①} \\ 3x+4y+bz=-1 & \text{--- ②} \\ 2x+10y+7z=c & \text{--- ③} \end{cases}$$

$$\text{①} \times 3 - \text{②} \Rightarrow 2y + (3a-b)z = 4$$

$$\text{①} \times 2 - \text{③} \Rightarrow -6y + (2a-7)z = 2-c$$

$$1^\circ \text{ 無限多解} \Leftrightarrow \frac{2}{-6} = \frac{3a-b}{2a-7} = \frac{4}{2-c}$$

$$\therefore \frac{1}{2}(2a-7) = \frac{-3}{-6}(3a-b) \quad \text{且} \quad 2-c = -12$$

$$\Rightarrow 2a-7 = -9a+3b \quad \text{且} \quad c=14$$

$$\Rightarrow 11a-3b=7 \quad \text{且} \quad c=14$$

$$2^\circ \text{ 無解} \Leftrightarrow \frac{2}{-6} = \frac{3a-b}{2a-7} \neq \frac{4}{2-c}$$

$$\Rightarrow 11a-3b=7 \quad \text{且} \quad c \neq 14$$

$$3^\circ \text{ 一解} \Leftrightarrow \frac{2}{-6} \neq \frac{3a-b}{2a-7}$$

$$\Rightarrow 11a-3b \neq 7$$

1) 有解 \Rightarrow 一解或無限多解 (x)

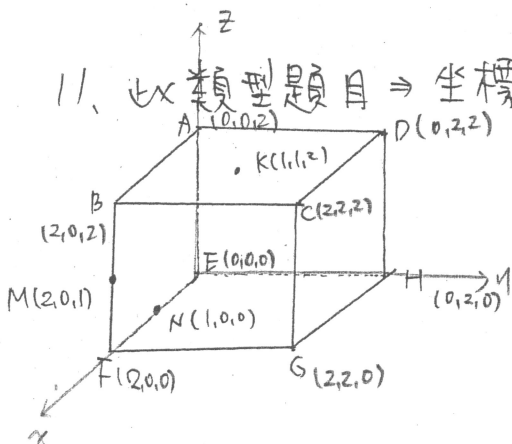
2) 有解 \Rightarrow $\begin{cases} \text{一解: } 11a-3b \neq 7 \\ \text{無限多解: } 11a-3b=7 \text{ 且 } c=14 \end{cases}$ (x)

3) 同 1 (x)

4) 無解 $\Rightarrow 11a-3b=7$ 且 $c \neq 14$ (o)

(4)(5) *

11. 此類題目 \Rightarrow 坐標化



$$1) \vec{KM} = (1, -1, -1)$$

$$\vec{AB} = (2, 0, 0)$$

$$\vec{AD} = (0, 2, 0)$$

$$\vec{AE} = (0, 0, -2)$$

$$\frac{1}{2}(\vec{AB} - \vec{AD} + \vec{AE}) = (1, -1, -1)$$

$$\Rightarrow \vec{KM} = \frac{1}{2}(\vec{AB} - \vec{AD} + \vec{AE}) \quad (o)$$

②) $\vec{KM} = (1, -1, -1)$, $\vec{AB} = (2, 0, 0)$
 $\vec{KM} \cdot \vec{AB} = 2 + 0 + 0 = 2$ (X)

③) $|\vec{KM}| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3}$ (X)

④) $\vec{KM} = (1, -1, -1)$
 $\vec{KN} = (0, -1, -2) \Rightarrow \vec{KM} \cdot \vec{MN} = (-1) + 0 + 1 = 0$
 $\vec{MN} = (-1, 0, -1) \therefore \angle M \text{ 為直角 } (0)$

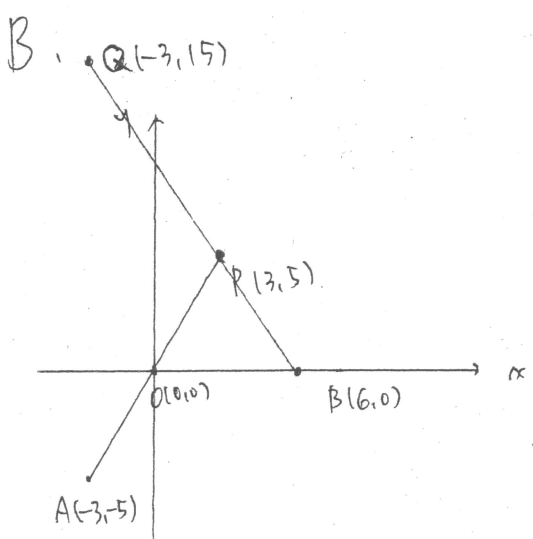
⑤) $\triangle KMN$ 由 \vec{KM}, \vec{KN} 所構成

1	-1	-1	-1
0	-1	-2	-1
1	2	-1	

$\therefore \triangle KMN = \frac{1}{2} \sqrt{1^2 + 2^2 + (-1)^2} = \frac{\sqrt{6}}{2}$ (X)

1) (4) #

- A.
- 99. \rightarrow 3 的倍數
 - 98 \rightarrow 2 的倍數
 - 97 \rightarrow 質數
 - 96 \rightarrow 2 的倍數
 - 95 \rightarrow 非 2, 3 倍數, 非質數
- 95 #



$\because \vec{AO}$ 前推至 P $\Rightarrow P(3, 5)$
 $\because \vec{PB}$ 前推至 Q $\Rightarrow Q(-3, 15)$
 $\because 3(\vec{AQ})$ 前推至 O $\Rightarrow \vec{AO} = 3(\vec{AQ})$
 $(3, -15) = 3(-3-x, 15-y)$
 $\Rightarrow (-3-x, 15-y) = (1, -5)$
 $\Rightarrow x = -4, y = 20$

(-4, 20) #

C. 設放入 x 顆其他色的球

事件 機率 報酬

藍色 $\frac{2}{x+7}$ 2000

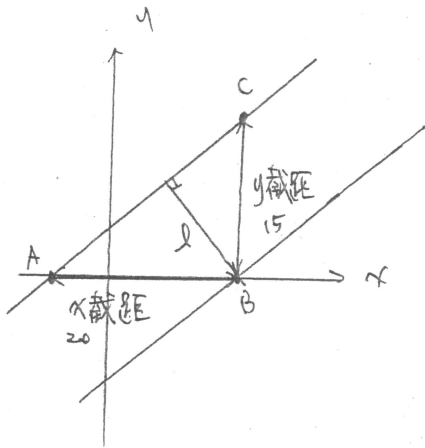
紅色 $\frac{5}{x+7}$ 1000

$$\Rightarrow \text{期望值} = \frac{2}{x+7} \cdot 2000 + \frac{5}{x+7} \cdot 1000 = 300$$

$$\Rightarrow \frac{4000}{x+7} + \frac{5000}{x+7} = 300 \Rightarrow 9000 = 300(x+7) \Rightarrow x+7 = 30 \Rightarrow x = 23$$

23 #

D.



設兩直線距離 l

由直角三角形 ABC 知 $\Rightarrow AC = 25$

$$\triangle ABC \text{ 面積} = \frac{1}{2} \cdot 15 \cdot 20 = \frac{1}{2} \cdot 25 \cdot l$$

$$\Rightarrow l = 12$$

12 #

E. 平方或絕對值函數內是對稱軸:

$$C_x: (y+2)^2 = 5x \Rightarrow y+2=0 \text{ 是對稱軸.}$$

$$|x|=4y \Rightarrow x=0 \text{ 是對稱軸.}$$

已知拋物線之對稱軸 $x = \frac{3}{4} \Rightarrow \text{設 } P_1: (x + \frac{3}{4})^2 = 4c(y - k)$

$$\text{又 } c = \frac{1}{8} \Rightarrow (x + \frac{3}{4})^2 = \frac{1}{2}(y - k)$$

$\therefore P_1, P_2$ 只有一個交點

$$\therefore \begin{cases} P_1 = 0 \\ P_2 = 0 \end{cases} \text{ 只有一組解} \Rightarrow \begin{cases} (x + \frac{3}{4})^2 = \frac{1}{2}(y - k) \text{ -- ①} \\ y = x^2 \text{ -- ②} \end{cases}$$

$$\text{①代入②} \Rightarrow (x + \frac{3}{4})^2 = \frac{1}{2}(x^2 - k) \Rightarrow x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{1}{2}x^2 - \frac{1}{2}k$$

$$\Rightarrow \frac{1}{2}x^2 + \frac{3}{2}x + (\frac{9}{16} + \frac{1}{2}k) = 0$$

$$\therefore \text{只有一解} \quad \therefore D = 0 \Rightarrow (\frac{3}{2})^2 - 4 \cdot \frac{1}{2} (\frac{9}{16} + \frac{1}{2}k) = 0$$

$$\therefore \frac{a}{4} - 2\left(\frac{a}{16} + \frac{1}{2}k\right) = 0$$

$$\Rightarrow 36 - 2(9 + 8k) = 0$$

$$\Rightarrow 36 - 18 - 16k = 0 \Rightarrow 16k = 18 \Rightarrow k = \frac{9}{8}$$

$$\frac{9}{8} \#$$

F. 類似問題 \Rightarrow 計算剩餘量.

設每年減少 $r\%$ \Rightarrow 每年剩 $1 - r\%$

設現在耐排放量 N .

$$\Rightarrow N \cdot (1 - r\%)^5 = 75\% \cdot N$$

$$\Rightarrow (1 - r\%)^5 = 0.75$$

$$\text{取 } \log \Rightarrow \log (1 - r\%)^5 = \log 0.75$$

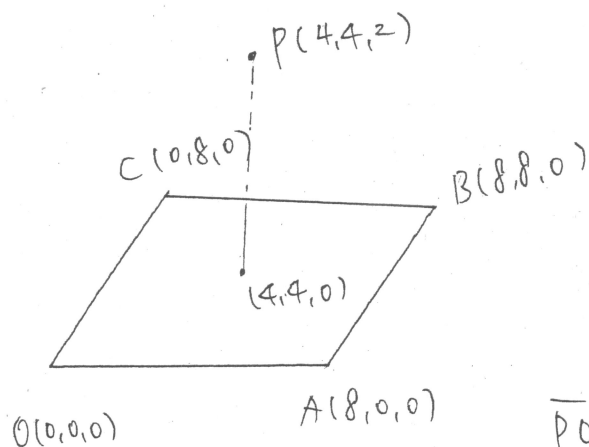
$$\Rightarrow 5 \log (1 - r\%) = \log \frac{3}{4} = \log 3 - \log 4 = -0.1249$$

$$\begin{aligned} \Rightarrow \log (1 - r\%) &= -0.02498 = -1 + 0.97502 \\ &= -1 + \log 9.44 \end{aligned}$$

$$\therefore 1 - r\% = 0.944 \Rightarrow r\% = 0.056 \Rightarrow r = 5.6$$

$$\underline{5.6} \#$$

G.



到四頂點等距

\Rightarrow P必落在ABCD之中心(4,4,0)

設 $P(4,4,z)$

$$\overline{PO} = 6 \Rightarrow \sqrt{4^2 + 4^2 + z^2} = 6$$

$$\Rightarrow 16 + 16 + z^2 = 36 \Rightarrow z^2 = 4 \Rightarrow z = \pm 2 \text{ (取正)}$$

找A, B, P平面. 求平面 \Rightarrow 找一點及法向量

$$\begin{aligned} \vec{n} &\perp \vec{AB} \\ \vec{n} &\perp \vec{AP} \end{aligned} \Rightarrow$$

$$\begin{array}{r} \begin{array}{cccc} 8 & 0 & 0 & 8 \\ -4 & 4 & 2 & -4 & 4 & 2 \end{array} \\ \hline 16 & 0 & 32 \end{array}$$

$$\vec{n} \parallel (16, 0, 32) \parallel (1, 0, 2)$$

$$\Rightarrow x + 2z = \underline{\underline{8}}$$

$$\vec{AR} = (0, 8, 0), \vec{AP} = (-4, 4, 2)$$

$$(b, c, d) = (0, z, 8) \# p. 7$$

H. 圓錐曲線 \Rightarrow 多考定義式

設橢圓之長軸長 $= 2a \Rightarrow \overline{PF_1} + \overline{PF_2} = 2a$
 短軸長 $= 2b$
 焦距 $= 2c$

又雙曲線之實軸長 $= 2b \Rightarrow |\overline{PF_1} - \overline{PF_2}| = 2b$

看到 $\overline{PF_1} \pm \overline{PF_2}$, 又有 $\overline{PF_1} \times \overline{PF_2} \Rightarrow$ 平方

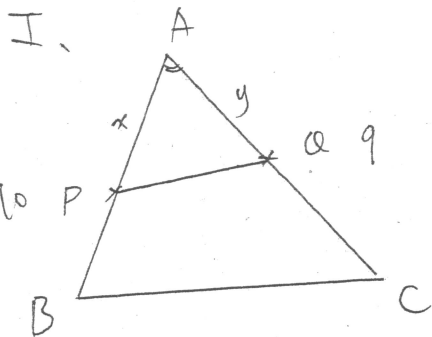
$$(\overline{PF_1} + \overline{PF_2})^2 = \overline{PF_1}^2 + \overline{PF_2}^2 + 2\overline{PF_1}\overline{PF_2} \Rightarrow 4a^2 = \overline{PF_1}^2 + \overline{PF_2}^2 + 2 \times 64 \quad \text{--- ①}$$

$$(\overline{PF_1} - \overline{PF_2})^2 = \overline{PF_1}^2 + \overline{PF_2}^2 - 2\overline{PF_1}\overline{PF_2} \Rightarrow 4b^2 = \overline{PF_1}^2 + \overline{PF_2}^2 - 2 \times 64 \quad \text{--- ②}$$

$$\text{①} - \text{②} \Rightarrow 4(a^2 - b^2) = 4 \times 64 \Rightarrow a^2 - b^2 = 64 \quad \text{又 } (a^2 = b^2 + c^2)$$

$$\therefore c^2 = 64 \Rightarrow c = 8 \Rightarrow 2c = 16$$

16 #



設 $\overline{AP} = x, \overline{AQ} = y$

$$\triangle ABC = \frac{1}{2} \cdot 10 \cdot 9 \cdot \sin A$$

$$\triangle APQ = \frac{1}{2} \cdot x \cdot y \cdot \sin A$$

$$\triangle ABC = 2 \triangle APQ \Rightarrow 90 = 2xy \Rightarrow xy = 45$$

求 $\overline{PQ} = \sqrt{x^2 + y^2 - 2xy \cos A} = \sqrt{x^2 + y^2 - 2 \cdot 45 \cdot \frac{3}{8}} = \sqrt{x^2 + y^2 - \frac{135}{4}}$

求極值均先不管常數 \Rightarrow 已知 $xy = 45$, 求 $x^2 + y^2$ (算幾不等式)

$$\Rightarrow \frac{x^2 + y^2}{2} \geq \sqrt{x^2 y^2} \Rightarrow \frac{x^2 + y^2}{2} \geq xy = 45 \Rightarrow x^2 + y^2 \geq 90$$

$$\therefore \overline{PQ} \geq \sqrt{90 - \frac{135}{4}} = \sqrt{\frac{225}{4}} = \frac{15}{2}$$

$\frac{15}{2}$ #